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AMERICAN GEOPHYSICAL UNION

TRANSACTIONS OF 1940

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REGIONAL MEETING

COLUMBUS, OHIO, DECEMBER, 1939

SYMPOSIA UNDER AUSPICES OF THE
AMERICAN ASSOCIATION FOR THE ADVANCEMENT
OF SCIENCE
(A) APPLICATIONS OF MATHEMATICS IN THE
EARTH-SCIENCES
(B) HYDROLOGIC PROBLEMS IN THE OHIO AND
MICHIGAN BASINS

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AMERICAN GEOPHYSICAL UNION

REGIONAL MEETING
AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

December 1939

PART IV-A

SYMPOSIUM ON APPLICATIONS OF MATHEMATICS IN THE EARTH-SCIENCES

Columbus, Ohio
December 29, 1939

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SYMPOSIUM ON
APPLICATIONS OF MATHEMATICS IN THE EARTH-SCIENCES

[A symposium presented under the joint auspices of Section A (Mathematics) and Section E (Geology and Geography) of the American Association for the Advancement of Science, the American Mathematical Society, the Mathematical Association of America, and the Geological Society of America, at Columbus, Ohio, December 29, 1939]

PREFACE

W. D. Cairns

The group of addresses given at the joint session of Sections A and E of the American Association for the Advancement of Science at Columbus, Ohio, in December 1939 is a happy outcome of the joint session of these sections at the Richmond meeting in December 1938. It tends to further a very desirable rapprochement between geophysicists and mathematicians, for the group represents informed men from both these fields and begins the important task of bringing to each field the knowledge or some suggestions of the contributions which the other field may make.

Mr. Lambert sketches the essentials in the development of gravitational geophysics in Great Britain, beginning with Newton in 1687. He discusses not so much the mathematical methods as the men and their results, although the former would have been of interest. It is an absorbing study to note the stumbling and disparate attempts to investigate the density of the Earth, the gravitation-constant, the elastic properties of the Earth, and the figure of the Earth including the related topics of the measurement of a degree of latitude, the determination of the geoid, and the variation in gravity. One may well contrast this slow development with the possibilities of the present day, where there is an increasing amount of cooperation, not merely between the various divisions in the field of geophysics but between geophysics and mathematics.

Professor Gilchrist and Mr. Clark apply standard mathematical processes to the measurement of magnetic and electric anomalies due to the presence of deposits of minerals which modify the magnetic intensity and the electrical conductivity, respectively. He exhibits clearly the theoretical considerations, their adaptation to the geological problem and the consequent interpretation of results. The interplay of theory and practice in this investigation is a good example of applied physics.

Doctor Slotnick gives an account, largely descriptive, of the two outstanding methods used in geophysical exploration for petroleum. He supplies the necessary formulas but, as is proper in a paper of this character, without developing these mathematically. Enough is given to suggest to the mathematician the character of the mathematical theory involved and its application to the field-problem; happily this includes statements as to the size of the quantities involved and the accuracy demanded in the measurements, as well as a critical comparison of the two methods. Could this group of papers have been presented with greater fullness, it would have been of decided value to the general reader if Doctor Slotnick had, on the one hand, indicated rather briefly the derivation of the mathematical formulas, with bibliographical references, and, on the other hand, supplied something concrete as to the actual computational methods and more specific examples of results of such investigations, divested of any commercial secrets. The parametric forms for x and t are of special interest to mathematicians, both from the gain in knowledge as to the nature of the functions and in the greater ease in computation.

Doctors Blake and Byerly map out lines of investigation which may well occupy the talents of able mathematicians and seismologists to the full extent of their time and thought. In no field, it seems to me, is there such a challenge to conquer the complexities of natural phenomena. The many unanswered questions and unsolved problems raised in these papers might well bring discouragement to scientists were it not for our knowledge of the great advances made in the past two or three decades. Theorists have "missed the bus," their theories have become discredited, but there is a steady, even if slow, progress toward the unification of seismological theory.

Doctor Blake's statement that it should be possible to design a general-purpose low-cost seismograph, with variable period, magnification and damping, adapted to almost any site is a challenge to the numerous men in Canada and the United States who are well prepared to design an instrument with these specifications. That a general-purpose instrument shall be capable, as he desires, of yielding quantitative information about the amplitudes of the earth-movements is a more drastic specification; it is one of the large defects in seismology that there is not a sufficient reliability in the measurement of amplitudes of the earth-movements. Even where some seismologists minimize the importance of mere amplitudes in favor of other characteristics obtainable from seismograms, it has not yet been proved, in my judgment, that such results are sufficiently accurate and reliable, as tested, for example, by diagrams of the components of the same earth-movements obtained by different methods or from different instruments. This is not the place, however, to develop this criticism.

Only by stating explicitly their perplexities can geophysicists expect to enlist the aid of mathematicians, and only through the criticisms of competent mathematicians can the validity of applications of mathematics to geophysical problems be established. Such meetings as here represented cannot help being of great potential value by bringing the efforts of workers in these two fields into a common pursuit of many outstanding problems in geophysics.

Oberlin College,
Oberlin, Ohio

THE BEGINNINGS OF MATHEMATICAL GEOPHYSICS IN GREAT BRITAIN

Walter D. Lambert

The title of this paper does not satisfactorily describe the paper itself--in fact, it would be hard to devise a brief and appropriate title. One rather limited aspect of the subject may appear to receive undue emphasis because that aspect illustrates how an important idea may remain undeveloped for a long while, even though the knowledge needed to develop it may be available. Nevertheless, nothing much seems to happen until the time for development becomes ripe.

This brief historical sketch deals mainly with large-scale, gravitational geophysics, for the good reason that mathematical geophysics began with problems of this kind. The problems dealt with were on a large scale, usually concerned with the Earth as a whole; gravitation was the first of the great forces of nature to yield to mathematical treatment.

Gravitational geophysics begins in the seventeenth century with the discovery of the law of gravitation by Sir Isaac Newton. The first edition of his "Principia" was published in 1687. In it he discusses several important general theorems on attractions. These he applies to determine the flattening of a rotating body having the size, rotation, and gravitational attraction of the Earth, on the supposition that it is fluid.

Remember that there was no science of hydrostatics, that Newton had to develop his integral calculus as he went along, and also that, in order to conform to prevailing concepts or fashions in mathematics, Newton had to state his results in what seems to us a forced and outlandish way. Whether or not it appeared forced and outlandish to his contemporaries, it must have caused him additional difficulty.

In spite of these handicaps Newton reached a substantially correct result--the flattening of his model of the Earth is 1/230. The difference between this result and the actual flattening is due to the incorrect assumption of homogeneity. It can not be said that his reasoning is rigorous, but the result is correct and represents a great intellectual achievement.

Newton also discussed the tides and had a correct conception of the tide-producing forces. Some of his attempts at explaining the curious phenomena connected with tides are less happy, but the same might be said of the efforts of many of his successors in this field of endeavor down to our own time.

The third edition of the "Principia", the final one in Newton's lifetime though edited by another hand, was not published until the eighteenth century, 1726. Newton died the next year. A little later, around 1740, Newton's ideas about the figure of the Earth were taken up, clarified, and developed by three Scots--Simpson, Stirling and Maclaurin--but the leadership in studies of this sort passed to France. It was not until about a century ago that this particular problem, that of the shape and attraction of a large, self-attracting, rotating, fluid, nearly spherical body, attained approximately its present stage of development. It is perhaps arbitrary to set a limit of this sort, for work in allied problems continued in the nineteenth century and still continues.

It might be mentioned that Maclaurin, perhaps the best known of the three, noticed the deflecting force of the Earth's rotation on a body moving relatively to Earth, but only for a north-south motion. The effect of rotation for the case of vertical motion had already been noted. He suggested that forces of this sort might affect winds and currents.

Another line of investigation, the determination of the mean density of the Earth, was undertaken by Maskelyne, Astronomer Royal, and by James Hutton, another Scot. The method used was to determine, by astronomical observations on a known mass, the deflections of the plumb-line caused by that mass. The mass used was that of the Mountain Schehallien in Perthshire, Scotland. Charles Mason--the same Mason who helped run Mason and Dixon's line--chose the astronomical stations, and Maskelyne made the observations.

Hutton, on the basis of a minute survey of the Mountain, divided its mass into 960 slender vertical prisms for the purpose of computing its attraction at the astronomical stations. It was found that, to make the observed attraction equal to the computed attraction, the mean density of the Earth must be 4.5, taking that of the Mountain as 2.5.

Towards the end of the century, in 1798, Henry Cavendish, by the experiment that bears his name, dealt with the attraction of known masses of laboratory dimensions, instead of the dimensions of a mountain, and determined the mean density of the Earth. Parenthetically it should be noted that the work of both Hutton and Cavendish and their successors may be described as a determination of either the mean density of the Earth or of the Newtonian gravitation-constant (6.66×10^{-8} cgs unit). The fact is that the product of the mean density and of the gravitation-constant is known with considerable accuracy; hence if we determine by experiment one factor of the product, we thereby determine the other factor also. The gravitation-constant is, as a matter of fact, the quantity that is determined experimentally.

The question of the effect of mountains, ocean-basins, and of all the minor irregularities of the Earth's surface on the direction of the plumb-line and on the intensity of gravity had been more or less in the minds of geodesists for many years. The expedition to Peru, sent out (1736-43) by the Paris Academy of Sciences to help decide whether the Earth is an oblate or prolate spheroid, encountered that question. There, if anywhere, among the immense mountain-masses of the Andes, the effects of surface-irregularities must be manifest. Bouguer observed the effect both on the plumb-line and on gravity. It was nowhere near as great as would be expected from the masses of the mountains; but, since the gravitation-constant or, if you prefer, the mean density of the Earth, was guessed at rather than known, it could not be determined just how much the reality of the gravitational effect fell short of what might be expected from the appearance of the Earth's surface. To explain the result Bouguer speculated on the possibility that the mountains might contain immense caves, thus giving from the outside an exaggerated impression of their mass.

Bouguer did not go beyond this, but less than a generation later another geodesist, Father Roger Joseph Boscovich, did. In 1750 le Maire and Boscovich began to measure a degree of latitude in the Papal States. Their account of the operation was published in 1755. Boscovich had also written extensively on problems in attraction. Referring to the results found by Bouguer and la Condamine in Peru he says [see Todhunter, A history of the mathematical theories of attraction and the figure of the Earth from the time of Newton to that of Laplace, London, 1873, v. 1, p. 313]:

> "Indeed most mountains, as it seems to me, are formed when the strata nearest the surface swell by the force of internal heat; if indeed this is the way of it, then no matter is added there and the empty space in the interior [Boscovich says 'among the innards', a literal translation, but not quite in keeping with his rather elaborate Latinity] compensates all that apparent mass of matter that rears itself up into the mountain."

This sounds very like the modern conception of isostasy, except that the type of compensation is hardly that of Pratt or Airy.

Let us turn to a spot nearer home, the Eastern Shore country, where Maryland and Delaware adjoin. There, in 1768, the experts, Charles Mason and Jeremiah Dixon, had been engaged jointly by the colonies of Maryland and Pennsylvania to lay down on the ground the common boundary as decided by long litigation and negotiation. The boundary-line along the parallel of 39° 43' is usually thought of as the Mason and Dixon Line, but it does not concern us here. What is now Delaware was then under the government of the Penn heirs, though with a separate legislature. The demarcation of the Maryland-Delaware Line was therefore also part of Mason and Dixon's job.

When Mason and Dixon noted the absence of rugged topography on the Eastern Shore, it occurred to them that it would be a fine region for geodetic operations to determine the figure of the Earth. They therefore applied to the Royal Society of London for a grant to cover expenses. Boscovich also is said to have urged upon the Royal Society the desirability of supporting Mason and Dixon's project. [The anonymous translator into French of one of Boscovich's works says, according to Todhunter on page 333 of the reference above:

> "Finally while in England he represented to the Royal Society the advantages of measuring a degree in America, and all the more because England had done nothing, since the introduction of improved methods in astronomy, toward ascertaining the figure of the earth."

The translator's name is given elsewhere, though not in the translation itself. The version appears to have been made with Boscovich's approval, if not cooperation.]

The Royal Society approved and, after the boundary-demarcation had been completed, they remeasured with special care the Delaware-Maryland Boundary, which makes an angle of less than 4° with a meridian. The astronomical observations had already been made in connection with the boundary-operations.

Maskelyne wrote up the operation in the Philosophical Transactions. He first says in his introduction to Mason and Dixon's own account [Phil. Trans., v. 58, p. 273]:

> "It may not be improper to remark, that the level disposition of the country this degree passes through, which, as I understand, also obtains further to the south, and, in a great measure to the north of the limits of the same, gives some advantage to this measure, with respect to the use that may be made of it in inquiring into the figure of the Earth; as there is no room for suspicion that the plumb-line of the sector [astronomical instrument used for determining the latitude] could be deflected materially by the attraction of any mountain, or even elevated ground of a more moderate height, continued for a great length; which latter circumstance, not taken notice of before [the same volume of the Transactions (p. 15) contains a reference to the attraction of mountains as a source of error in geodetic operations; it is a letter from Father Liesganig describing work in Austria (Moravia-Croatia)], the learned Father Boscovich has shown may produce a very considerable deviation of the plumb-line, in the elaborate treatise of the measure of a degree of the meridian between Rome and Rimini taken by himself and his learned coadjutor, Father le Maire."

But in his postcript to Mason and Dixon's account, Maskelyne is obliged to admit that local smoothness of topography is not so important [see Phil Trans., v. 58, p. 328, and also Todhunter, v. 1, p. 448; the latter supplies the handle "The Honourable" to Cavendish's name--Cavendish was the grandson of a duke and a very odd genius indeed]:

> ".....But the Honourable Mr. Henry Cavendish has since considered this matter more minutely; and having mathematically investigated several rules for finding the attraction of the inequalities of the Earth, has, upon probable suppositions of the distance and height of the Allegany Mountains from the degree measured, and the depth and declivity of the Atlantic; computed what alteration might be produced in the length of the degree, from the attraction of the Atlantic; and finds the degree may have been diminished by 60 or 100 toises from these causes. He has also found, by similar calculations, that the degrees measured in Italy, and at the Cape of Good Hope, may be very sensibly affected by the attraction of the Mediterranean Sea and Indian Ocean."

Cavendish is quite right, and the advantages of level country set forth in the first quotation from Maskelyne are there overestimated.

Todhunter adds, after the quotations just given, the following:

> "Frisi, in his Cosmographia, Vol. II, page 91, seems to think that this diminution of 60 or 100 toises in the American arc can hardly be accepted. It is indeed difficult to see how the Atlantic Ocean can have produced any appreciable effect. Bailly refers to the passage without any expression of dissent in his 'Histoire de l'Astronomie Moderne', Vol. III, page 41."

Todhunter, too, is wrong. He probably thought of the Atlantic Coast as running due north and south, so that the oceanic depression and the continental rise would produce but little effect on the latitude, whatever the effect might be in longitude and azimuth. But the southwesterly trend of the coast would give a rather large deflection in latitude, and a deflection in the prime vertical, which affects longitude and azimuth, not very much larger. Hayford [J. F. Hayford, The figure of the Earth and isostasy from measurements in the United States, Gov. Ptg. Office, 1909, pp. 48-51] gives the figures for various points in this general region. The computed deflection in latitude for the topography alone is about 30" and in longitude 40" or more. Because of isostatic compensation, the computed isostatic deflections are much less. The isostatic deflection in latitude varies but little from point to point in this region. In the case of Mason and Dixon's arc it would be the difference of the deflections that would affect the length of a degree. Cavendish's estimated effect of 60 toises (360 French feet, or about 4" in latitude) may therefore be exaggerated, although we now know from experience that actual (not computed) deflections of about 4" or more may very easily occur without there appearing in the surface-conformation of the ground any sufficient reason for such a deflection. Deflections much larger than 4" and not explainable by surface-topography are not uncommon. It would be interesting to make the computation for the two ends of Mason and Dixon's arc, also to reobserve the latitudes. Two thoughts are suggested by this narrative. The first is the obvious one: Do not guess, but calculate, especially if there are no calculations of the same sort available, and if everyday experience is untrustworthy. The second thought is, why was not all this done much earlier?

Even in Bouguer's time the integral calculus was sufficiently developed to provide the necessary formulas. Part of the failure to make the computations, the need for which now seems obvious, may have been because of ignorance of the mean density of the Earth and of the detailed configuration of continents and oceans.

But it seems to me that the chief reason was the labor involved in making such computations. Men of science were relatively few, and means at their disposal were small in comparison with those available today. They were not numerous enough to do much scientific drudgery themselves or rich enough to hire it done. A step in advance may seem possible, and we wonder why it was not taken. The answer seems to be that further technical or general development was needed. In short, the time was not ripe.

Towards the close of the eighteenth century, namely in 1798, Cavendish published his determination of the mean density of the Earth. He found it 5.48 as against 4.5 found from the attraction of Schehallien. Cavendish's result is close to the now accepted value. He states in his memoir that the experiment was made with the apparatus of the Rev. John Michell, who "many years ago . . . ., contrived a method of determining the density of the Earth, by rendering sensible the attraction of small quantities of matter." Michell died soon after he completed the apparatus. The man who next had it was a professor of natural philosophy, or as we would now say, "of physics," at Cambridge. In spite of his position he had no facilities for experimental work and so passed the apparatus on to Cavendish.

The first part of the nineteenth century need not detain us long. For the first few decades the leadership was clearly in continental, principally in French, hands. Just before the middle of the century (1849) Stokes published a memoir "On the variation of gravity at the surface of the Earth" [originally published in Trans. Cambridge Phil. Soc; republished in Stokes's Mathematical and Physical Papers, Vol. II, Cambridge Univer. Press, 1883, p. 131]. It gives precision to concepts that had been understood only vaguely or not at all. Some of these concepts are still pitfalls for the unwary. He also gave a formula for determining the figure of the Earth from gravity-observations. This means more than determining the general figure of the Earth, that is, the flattening, from gravity-observations. That had been theoretically possible for a century before the time when Stokes wrote, though sufficient observations were lacking until near Stokes's own time. What is meant is the figure of the Earth in detail, the "humps and hollows of the geoid," as we say with more picturesqueness than accuracy. These so-called humps and hollows are in fact, mere modifications of the curvature, for even in a "hollow" the geoid is still convex. [Theoretically hollows of very small extent and depth are possible, but their practical importance is nil.]

This is of interest not merely for large-scale geophysics but for small-scale, local geophysics also. If we know the inequalities of the geoid, we know where the excesses and deficiencies of matter are located, so far as effective amount and horizontal position are concerned. This is important information for the study of crustal structure.

Unfortunately the application of Stokes's formula for the elevation or depression of the geoid at any point requires the knowledge of gravity all over the Earth. In Stokes's time, this ideal state of affairs was nowhere near realization. Now, almost a century later, we are still far from having really adequate data, but are looking forward to the time when we shall have. Meanwhile we are making tentative applications of Stokes's formula, supplementing our ignorance with guesswork. Even these somewhat hypothetical inferences are helpful.

So much has been said elsewhere about isostasy in connection with Pratt and Airy that I shall add nothing here but shall pass on after remarking that Airy made a notable contribution to the theory of tides and waves. Stokes also contributed to the theory of wave-motion.

We now come to relatively modern times, and the difficulty lies in knowing what to discuss. The field of mathematical geophysics is immensely broadened. Problems not arising immediately out of the law of gravitation are discussed, also problems in small-scale or local geophysics.

Among the great names are Kelvin, Tait, Darwin, to mention only those who are dead. Some of the new problems are those of the internal heat of the Earth, empirical prediction of tides, the elastic properties of the Earth, earth-tides, tidal friction, variation of latitude. To all of these, British geophysicists have made substantial contributions.

Mathematical geophysics flourishes today in Great Britain, and it may be of interest to consider why. For one thing, there has been a center of interest in the subject at Cambridge University. Beginning with Newton, or perhaps even earlier, Cambridge has devoted considerable attention to the mathematical treatment, not only of geophysical problems, but of problems in natural science as well. Lately there has been a journal especially devoted to mathematical geophysics, namely, The Geophysical Supplement to the Monthly Notices of the Royal Astronomical Society. The papers in this journal are largely mathematical and of a high order of excellence. Finally, Great Britain is a compact country, and opportunities for personal intercourse among geophysicists are better than in the wide-spread area of our country.

We on this side of the Atlantic might found a journal and, presupposing the unpredictable endowment of mathematical ability, we might develop a tradition of mathematical research in problems of geophysics, but the handicap of distance would remain. We may expect, when the time is ripe, to have a considerable development of mathematical geophysics in this country, too.

U. S. Coast and Geodetic Survey,
Washington, D. C.

THE USE OF MATHEMATICS IN THE DELINEATION OF MAGNETIC AND ELECTRIC ANOMALIES

Lachlan Gilchrist and A. R. Clark

(I) The delineation of magnetic anomalies

There appear to be two classes of magnetic anomalies--herein designated as (A) and (B).

(A) Anomalies in which a large but finite number of consequent poles are to be found in the subsurface material, and for which there is a resultant equivalent magnet--This resultant magnet may have many forms, but in many cases a magnet of the form of a rectangular parallelopiped may be taken as the approximately equivalent magnet. Numerous deposits of magnetite, pyrrhotite, ilmenite, franklinite, and magnetic dikes are found to provide this condition. Moreover, it is frequently found that the magnetic meridian plane of these bodies has only a small declination from the Earth's magnetic meridian plane.

(1) If the magnetic body is extensive in the direction perpendicular to its magnetic meridian plane (X, Z), the value of the vertical component ΔZ and the horizontal component, ΔH, of the field-intensity of the body at points (x,0) on the surface of the ground above its median section

are given by the formula

$$\Delta Z = \frac{\rho}{d}\left[\frac{x + a}{\left\{(x + a)^2 + d^2\right\}^{1/2}} - \frac{x - a}{\left\{(x - a)^2 + d^2\right\}^{1/2}}\right]$$

$$+ \frac{\rho}{(d + \ell \sin \alpha)}\left[\frac{x - a - \ell \cos \alpha}{\left\{(x-a-\ell \cos \alpha)^2 + (d + \ell \sin \alpha)^2\right\}^{1/2}} \quad \frac{x + a - \ell \cos \alpha}{\left\{(x+a-\ell \cos \alpha)^2 + (d + \ell \sin \alpha)^2\right\}^{1/2}}\right]$$

$$\Delta H = \rho\left[\frac{1}{\left\{(x + a)^2 + d^2\right\}^{1/2}} - \frac{1}{\left\{(x - a)^2 + d^2\right\}^{1/2}}\right]$$

$$- \rho\left[\frac{1}{\left\{(x + a - \ell \cos \alpha)^2 + (d + \ell \sin \alpha)^2\right\}^{1/2}} - \frac{1}{\left\{(x - a - \ell \cos \alpha)^2 + (d + \ell \sin \alpha)^2\right\}^{1/2}}\right]$$

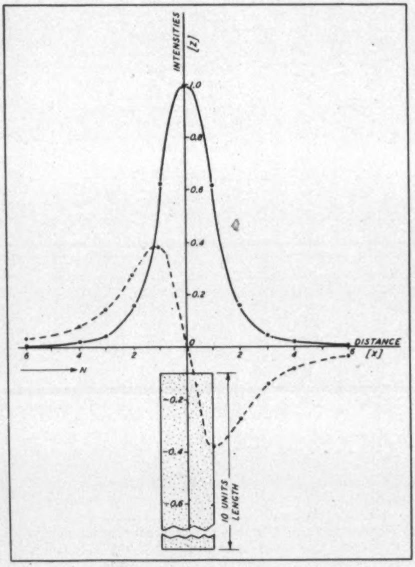

Fig. 1--

Graphs of vertical and horizontal intensities due to ideal magnets

where ρ = the average intensity of magnetization, d = depth to the upper pole, α = angle of dip, a = 1/2 width, ℓ = length or distance from the upper to the lower pole, and x = distance from the origin which is taken directly above the mid-point of the top of the magnet.

If measurements of the vertical and horizontal components of the field-intensity due to the body are made at well-chosen points on the surface of the ground, and contours of the field-intensities are mapped, the magnetic meridian plane due to the body in its meridian section may be determined approximately. If now the measured values of the vertical and horizontal components in this median section are plotted as shown in Figure 1, the values of a, d, ℓ, and x may be determined and the value of ρ, the intensity of magnetization, may be calculated from the formulae which are presented if measured values of ΔZ and ΔH taken at suitable points are inserted in the formulas.

(2) If the magnetic body is not extensive in the direction at right-angles to its magnetic meridian plane, the following formula may be taken to represent the vertical component intensity at the point on the surface where it is a maximum, namely, ΔZ_{max}

$$\Delta V = \Delta Z_{max} = 2\rho \tan^{-1}\left(\frac{ba}{d}\right)\sqrt{\frac{1}{d^2 + a^2 + b^2}} - 2\rho \tan^{-1}(-b'a)\sqrt{\frac{1}{d^2 + a^2 + (-b')^2}}$$

$$+ \rho \tan^{-1}\left(\frac{bN}{M}\right)\sqrt{\frac{1}{M^2 + N^2 + b^2}} - \rho \tan^{-1}\left(\frac{-b'N}{M}\right)\sqrt{\frac{1}{M^2 + N^2 + (-b')^2}}$$

$$- \rho \tan^{-1}\left(\frac{bO}{M}\right)\sqrt{\frac{1}{M^2 + O^2 + b^2}} + \rho \tan^{-1}\left(\frac{-b'O}{M}\right)\sqrt{\frac{1}{M^2 + O^2 + (-b')^2}}$$

where d = depth to the upper pole at the point of maximum intensity, 2a = width of the magnet,

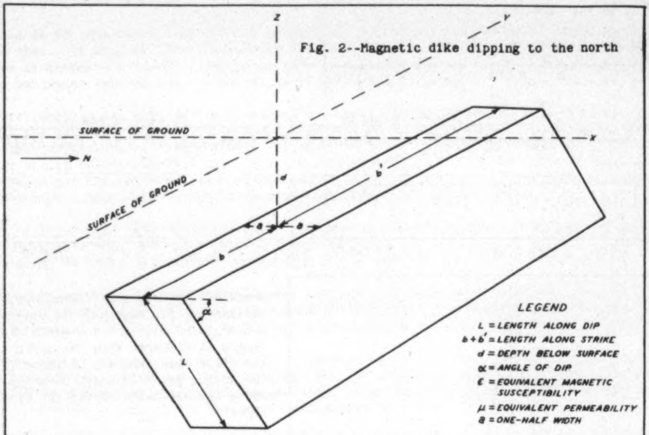

Fig. 2--Magnetic dike dipping to the north

District	d	α	2a	L	b	b'	V	I	V_0	$\epsilon = (I/V_0)$	$\mu = (1+4\pi\epsilon)$
	feet	°	feet	feet	feet	feet	Γ		Γ		
1 Falcon Bridge	133	80N	132	700	150	150	.0142	.0107	.591	.0180	1.23
Lot 11, concession 4	130	80N	130	650	90	120	.0142	.0127	.591	.0213	1.27
2	100	45S	50	300	300	300	.0160	.0201	.610	.0329	1.41
3	50	45N	50	200	200	200	.0500	.0303	.610	.0497	1.62
4	100	45S	75	300	400	600	.0604	.0525	.610	.0860	2.11
5	100	40S	75	300	100	300	.0500	.0489	.610	.0800	2.00

(b + b') = length of the magnet along the direction perpendicular to the magnetic meridian plane, L = distance from the upper to the lower pole, I = ρ = intensity of magnetization, V_0 = normal vertical intensity for the region, $\xi = (\rho/V_0)$ = equivalent magnetic susceptibility, α = angle of dip, and where for a body dipping south $M = -d - L \sin \alpha$, $N = a - L \cos \alpha$, $O = -a - L \cos \alpha$, and for a body dipping north $M = -d - L \sin \alpha$, $N = a + L \cos \alpha$, $O = -a + L \cos \alpha$.

As this vertical component intensity may be measured, and as the depth and dimensions of the equivalent magnet may be obtained approximately from the contours of the measured values of ΔZ and ΔH, the value of the average intensity of magnetization, ρ, may be calculated. If the values of ρ obtained for several bodies under exploration are tabulated, the investigator may at least distinguish deposits of massive magnetite from the other materials and, with the assistance of other geophysical evidence, may distinguish magnetic dikes of rock from magnetic bodies such as pyrrhotite, ilmenite, and franklinite.

From the values of ρ and the normal value of the vertical intensity in the field, V_0, an hypothetical susceptibility ξ may be obtained from $\xi = (\rho/V_0)$ and the permeability, μ, may be calculated from $\mu = (1 + 4\pi\xi)$. The values of ρ, ξ, and μ for the materials in several deposits are presented in Figure 2. The comparatively small value of V_0 and large values of ρ and ξ immediately give rise to speculative questions on the justification of the hypothesis that the Earth's field acting on the material produced the magnetization of intensity ρ. For example, was the Earth's magnetic field-intensity much greater when these materials were formed on passing through a cooling process under pressure? Or was the Earth's magnetic field-intensity much as it is now, and were the high values of μ due to slow increase in the induction over a long time?

The following approximations and difficulties of investigation should be noted:

(a) In (A - 1), the magnitude of the dimensions d, a, are determined by well-known methods, but on the assumption that the magnetic body is extensive in the direction perpendicular to its magnetic meridian plane. If the body extends in this direction to distances greater than d or a, the approximate values of d, α, and a are quite satisfactory and the deduction of the order of magnitude of the intensity of magnetization, ρ, and the delineation of massive magnetite from pyrrhotite, ilmenite, and franklinite, or from disseminated magnetite in a basic dike, may be made.

(b) In (A - 2) similar approximations to those in (A - 1) exist, and the values of d, α, and a which are obtained by the usual methods may be only roughly approximate. As in the calculation of ρ only the value of the maximum vertical component of the field-intensity due to the body, ΔZ_{max}, is used, rough but useful delineation of the character of the magnetic body may be made.

(c) The actual problem for investigation in the field is found frequently to be more complex than that which has been considered, owing to the existence of groups of massive magnetic bodies of dimensions and distance of separation of the same order as their depth. The problem is rendered still more complex if disseminated magnetic material exists in the space between the magnetic bodies.

In order to cope with the difficulties indicated in (a), (b), and (c), it is of great assistance to plot the contours of ΔZ and ΔH and the graphs of the relative magnitudes of these quantities along and perpendicular to the magnetic meridian of numerous hypothetical individual magnetic bodies and groups of magnetic bodies. Well chosen examples of such typical cases have been found very useful indeed in the interpretation of the results of measurement in the field.

(B) Anomalies due to a difference in permeability to the Earth's magnetic field of subsurface masses of homogeneous materials extensive in the direction at right-angles to the magnetic meridian plane--An example is provided by a sedimentary deposit of massive crystalline dolomite in which is embedded porous dolomite containing about one-fifth of its volume of oil or salt water. If these materials are free from disseminated magnetic substances such as magnetite, then clusters of consequent poles will not exist; and if the dolomite body is of considerable depth, its effect in general on the magnetometer-needles placed on the surface of the ground will not be that of a finite equivalent magnet with upper and lower poles, but rather that of a homogeneous magnet extending downward to very great depth, in which no definite positions of finite resultant poles exist.

In this case, the vertical component Z and the horizontal component H of the field intensity at a point taken as origin in the meridian plane of the Earth's field directly above the center of an extensive body of material of rectangular section, are given by the formulas

$$Z = \frac{\mu V_0}{R^3[x \sin \alpha + d \cos \alpha]^2} [R^2 \cos \alpha (x + R \cos \alpha) + xd(x \sin \alpha + d \cos \alpha)]$$

$$H = \frac{\mu V_0}{R^3[x \sin \alpha + d \cos \alpha]^2} [d^3 \cos \alpha - x^3 \sin \alpha - R^3 \cos \alpha \sin \alpha]$$

where V_0 = normal vertical component of the Earth's field, μ = permeability, d = distance from the origin to the top of the body, x = coordinate perpendicular to d in the magnetic meridian plane, α = angle of dip, $R^2 = x^2 + d^2$. The form of the graphs of the Z- and H-values is given in Figure 3.

If the mass of porous dolomite has the form of a rectangular parallelopiped of permeability μ, the value of the vertical component of the magnetic field-intensity at a point on the surface of the ground directly above its center may be obtained approximately and fairly simply if the angle of dip α is 90°, and if its depth and dimensions are known. In an actual field-case, in which an extensive mass of porous dolomite containing oil and salt water was embedded in non-porous crystalline dolomite with its upper face at a depth of 1600 feet and extending downward to great depth, the difference in the magnitudes of the vertical component of the field-intensity at a point directly over the center of the embedded porous dolomite and of the value over adjacent non-porous dolomite was calculated to be about 20 γ. In this calculation the values of μ which were used are given in Table 1.

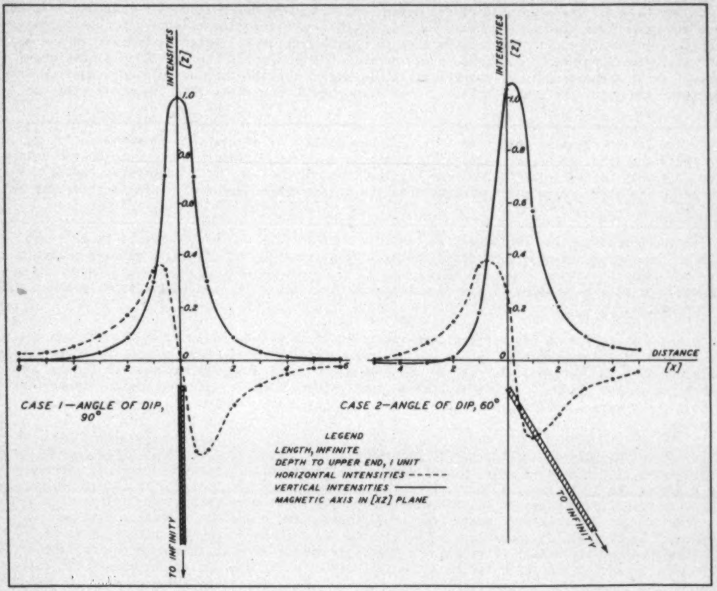

Fig. 3--Graphs of anomalies due to differences in permeability in homogeneous materials

Table 1--Magnetic permeabilities

Substance	Density	Permeability
Water (20°C)	1.00	0.99999056
Rock salt	2.15	0.99999893
Dolomite	2.90	1.00003622
Petroleum	0.90	0.99999062

The difference obtained by measurements in the field was about 100 γ. Since the difference was comparatively small it was necessary in course of measurement to take account of (1) the daily variation in the Earth's field-intensity, and (2) the change in the Earth's field-intensity per mile in a south-north direction. The results of the measurement are shown in Figure 4.

(II) The delineation of electric anomalies

Electrical conductivity was measured in the same region in which the magnetic measurements presented in (I-B) were made. The measurements of conductivity at depth were made by a modification of the lay-out used by the author in electrically logging a drill-hole in 1929, as described in Memoir 170 of the publications issued by the Geological Survey of Canada. The modified lay-out is presented in Figure 5 in plan and in elevation. It was believed that, from these conductivity-measurements, porous dolomite containing gas or oil could be distinguished from porous dolomite containing salt water and that, in conjunction with the results of the magnetic measurements, both of these could be distinguished from massive crystalline dolomite. Further, in conjunction with the results of conductivity-measurements made on the surface of the ground, it was hoped that some knowledge of the depth from the surface of oil or salt water could be obtained.

From Figure 5 it will be observed that the current point-electrode at the bottom of the drill-hole is a center of symmetry with respect to the line-electrodes on the surface, not only in space, but also electrically, for the stake-resistances all along the line-electrodes were equalized; but the electrical symmetry of the subsurface region about this central point will depend on the homogeneity of the materials. Moreover, the vertical plane through the central point-electrode is a central plane of symmetry with respect to the line-electrodes. Then, by

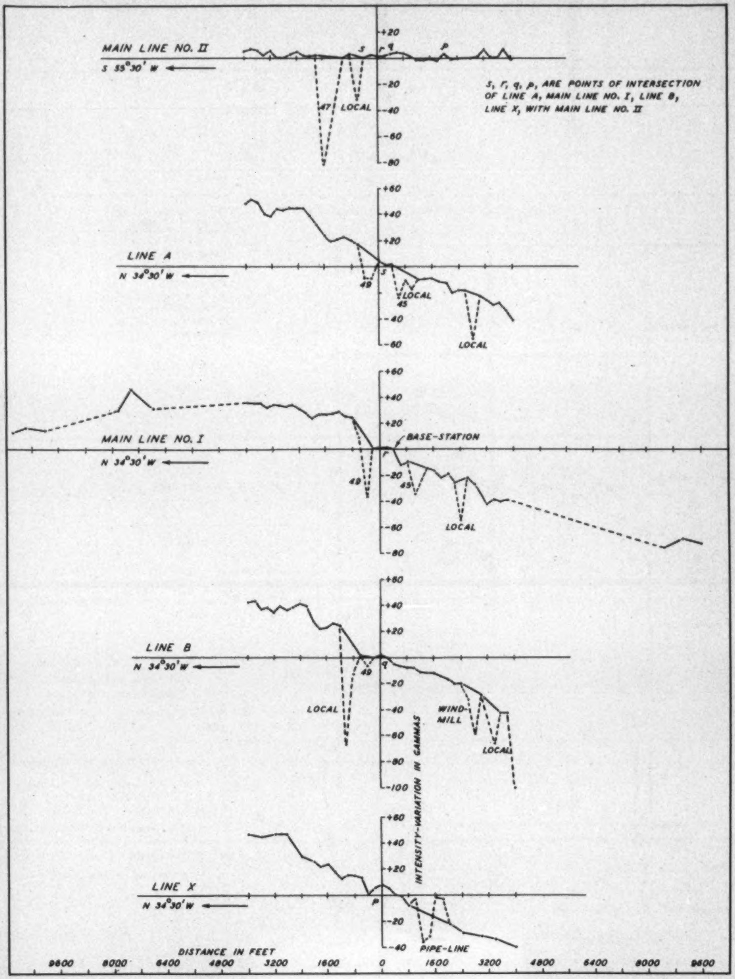

Fig. 4--Changes in vertical magnetic intensity from that at base-station, main line No. 1 (400 feet southeast), corrected for latitude-variation (15 gammas per mile--computed) and diurnal variation (measured)

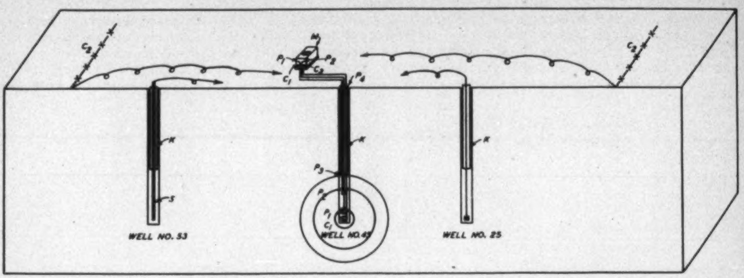
C_2
M
P_1
P_2
C_1
C_2
P_4
P_3
P_2
P_1
C_1
K
S
WELL NO. 53
WELL NO. 45
WELL NO. 25
C_2
SCALE IN FEET
0
800
1600
2400
3200
(SCALE DOES NOT APPLY TO ELECTRODES AND WELL-DIAMETER)
LEGEND
M = MEGGER
P_1, P_2 = POTENTIAL ELECTRODES
C_1 = CURRENT-POINT ELECTRODE
C_2 = CURRENT-LINE ELECTRODE
S = SIPHON
K = CASING

Fig. 5-A--Drill-hole lay-out

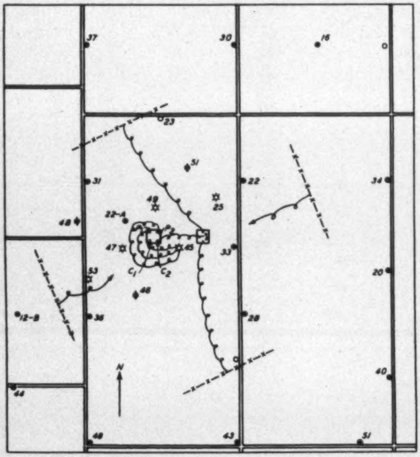
37
30
16
23
51
31
22
34
49
25
22-A
48
47
45
P_2
33
C_1
C_2
53
20
46
12-B
36
28
N
40
44
48
43
51
SCALE IN FEET
0
2000
4000
6000
LEGEND
= GAS-WELL
= CURRENT-ELECTRODES
= ELECTRICAL INSTRUMENTS
= TWO-WAY SWITCH

Fig. 5-B--Surface lay-out

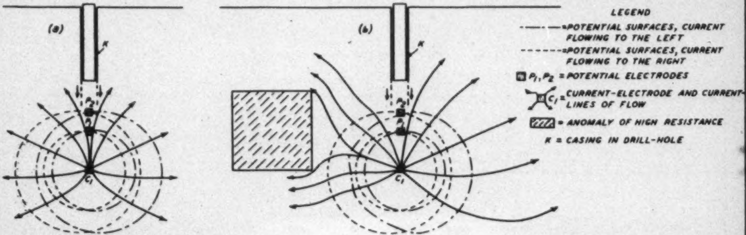
(a)
K
P_2
P_1
C_1
(b)
K
P_2
P_1
C_1
LEGEND
= POTENTIAL SURFACES, CURRENT FLOWING TO THE LEFT
= POTENTIAL SURFACES, CURRENT FLOWING TO THE RIGHT
P_1, P_2 = POTENTIAL ELECTRODES
C_1 = CURRENT-ELECTRODE AND CURRENT-LINES OF FLOW
= ANOMALY OF HIGH RESISTANCE
K = CASING IN DRILL-HOLE

Fig. 5-C--Current-flow and equipotential surfaces
(a) In a homogeneous medium
(b) Distorted by a high-resistance body

Fig. 5--Electrical set-up to determine subsurface resistances [A = drill-hole lay-out; B = surface lay-out; C = current-flow and equipotential surfaces (a) in homogeneous medium and (b) distorted by a high-resistance body

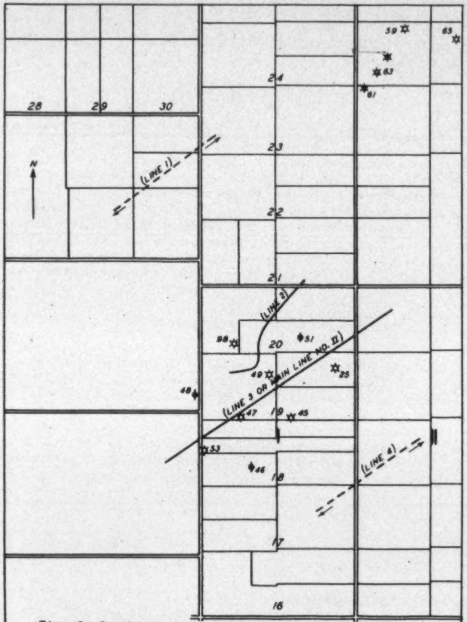

Fig. 6--Geophysical map of Dawn and Sombra townships, Lambton County, Ontario, September to October, 1938

sending the current from the central point to the line-electrodes in succession, the drop in potential between any two points in the central plane of symmetry is a comparative measure of the conductivity of the materials in either direction from the bottom of the drill-hole. If the four parallel-line current-electrodes are used in parallel as C_2 and the central-point current-electrode as C_1, then the approximate value of the average resistivity, ρ, of the materials in the neighborhood of the central point may be determined from

$$\rho = 4\pi R\left(\frac{ab}{b - a}\right)$$

where R is the interbowl resistance between two bowls of radii a and b, respectively. If the calculated value of ρ is below 10^5 ohms per centimeter cube, the presence of salt water is indicated. If considerable oil or gas or crystalline dolomite exists in the region, the value of ρ will be higher than 10^5 ohms per centimeter cube. The results of these sub-surface measurements indicated: (a) That low values of ρ existed in the region adjacent to the bottom of the drill-hole; (b) that the resistance in the west direction was slightly higher than in the east direction; and (c) that the resistance in the north direction was about 20 percent higher than in the south direction.

The lay-out for the measurement of the resistance on the surface was made by using the iron pipe-casing in the hole, 1,200 feet long, as central current-electrode, C_1. The results indicated: (a) That the resistance in the northwest direction was two to three times as high as in the southeast direction; and (b) that the high resistance in the northwest direction was due chiefly to materials less than 2000 feet in depth.

(III) Consideration of the formula used in the calculation of the specific resistivity

The formula used in the calculation of the specific resistivity, ρ, in the neighborhood of the bottom of the drill-hole 45, is

$$\rho = 4\pi R\left(\frac{ab}{b - a}\right) = 4\pi\left(\frac{{}_aV_b}{i}\right)\left(\frac{ab}{b - a}\right)$$

where ${}_aV_b$ is the interbowl potential-drop and i is the current passing through the ground.

As the equation of distribution of applied potential for a steady flow in a homogeneous conductor, namely, $(\partial^2V/\partial x^2) + (\partial^2V/\partial y^2) + (\partial^2V/\partial z^2) = 0$, implies continuity, and the solution of this equation leads to the formula for the specific resistivity, ρ, it is evident that the formula can apply only approximately to the conditions of flow in a non-homogeneous medium. However, the results of the measurement of R were obtained chiefly to make comparisons of the resistance in adjacent regions; and the measured resistancy R was used in the calculation of ρ only for the values of R in the immediate neighborhood of the bottom of the drill-hole in order to obtain a knowledge of the approximate value of ρ, from which the probable existence of saline water might be determined.

(IV) Consideration of the measurement of $R = ({}_aV_b/i)$

The applied potential was alternating, and very low frequencies of alternation were used, as well as a frequency of 500 cycles per second. In the use of the latter, care was taken to provide that self-inductance in the current-leads and mutual inductance in the potential-leads from the alternating current in the current-leads were negligibly small. It must be recognized, however, that portions of the Earth through which flows an electrical current form in reality a "leaky condenser" of finite capacity and, therefore, reactance as well as ohmic resistance must be taken into account. The measurement of ${}_aV_b$ was made by means of an alternating-potential potentiometer in which compensation for the reactance was made by means of a mutual inductance. The measurement of current was made by means of an alternating-current milliammeter. A reactance-free standard-resistance of known value was used as a comparator.

The value of the ohmic resistance in ohms was then calculated by means of the formula

$$R = (I_PE_P + I_QE_Q)/I^2$$

and the reactance in ohms was obtained from

$$x = (I_PE_Q - I_QE_P)/I^2$$

where I_P is the current in phase, I_Q is the current in quadrature, E_P is the EMF in phase, E_Q is the EMF in quadrature, $I = I_P + \sqrt{-1}\ I_Q$, and $E = E_P + \sqrt{-1}\ E_Q$. I_P and I_Q may be determined if the potential-drop across the known standard reactance-free resistance is measured.

(V) Conclusions

A study of the results of the magnetometric measurements in conjunction with the results of the measurements of electrical conductivity led to the following conclusions presented on Figure 6:

Area I (between Line 1 and Line 2)
Probably contains massive dolomite and may contain a small amount of gas

Area II (between Line 2 and Line 3)
Probably contains massive dolomite and may contain gas with the possibility of saline water and a small quantity of oil at the southern edge of the area

Area III (between Line 3 and Line 4)
Probably contains considerable saline water and possibly some oil near Line 3

The conclusion was also reached that, in the area between Line 1 and Line 2, very little saline water existed above a depth of 2,000 feet.

A drill-hole (No. 98) was put down at a point about 1,600 feet north and slightly west of drill-hole 49 (Figure 6), that is, in the area between Line 1 and Line 2. The depth of this hole was about 2,300 feet. The following results were found: (1) Massive crystalline dolomite existed under other materials to a depth of over 2,000 feet; (2) no saline water was found above a depth of 1,905 feet. It may be noted also that drill-hole 48 supplies a considerable quantity of gas.

Acknowledgments--The investigations which have been described were carried out in cooperation with M. J. S. Innes, K. C. Mann, and D. W. R. McKinley.

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GRAVIMETRIC AND SEISMIC METHODS IN PETROLEUM EXPLORATORY GEOPHYSICS

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Before 1885, persons engaged in the search for oil and gas held very erratic conceptions regarding the mode and cause of its accumulation. It was commonly supposed in the early days of the petroleum-industry that oil and gas lay in "pools" or "streams" beneath the ground.

In 1885, I. C. White promulgated his Anticlinal Theory [Science, 1885], which was the first scientific attempt to crystallize the ideas of geologists on the subject of the accumulation of petroleum. All subsequent search for petroleum has been based on this Anticlinal Theory or modifications of it.

Briefly enunciated for our purposes, the Anticlinal Theory states that the crust of the Earth has been folded by tectonic forces in such a manner as to furnish "traps" for the accumulation of oil and gas. Later it was found by experience that these folds more often contain petroleum-deposits if they occur in sedimentary basins, although petroleum-geologists are likely to view anticlines with interest even though they occur on broad structural arches or on the flanks of mountains. After it was found that oil and gas consistently migrate up-dip through porous reservoir-rocks and rise through the connate water in these rocks to the highest points under the anticline, it became apparent to petroleum-geologists that other types of structure in the Earth's crust could have the same effect as that produced by true anticlines. Therefore, in his list of possible traps, a geologist includes beds arched over igneous or salt intrusions, buried shorelines, faults which seal off porous rocks, and other similar "traps".

Because of the difference in specific gravities, the most common sequence found within the porous section beneath a structural trap is: Water at the lowest position; an accumulation of oil lying upon the water; and above the accumulation of oil, a space which is filled with gas, commonly known as a gas-cap. When a boring has been bottomed within that portion of the reservoir-rock containing oil in an accumulation of this ideal type, the hydrostatic pressure of the water below, the pressure in the gas-cap above, and the expanding gas in solution, all furnish the energy which ejects the petroleum from the porous rock and carries it to the well-head. When most of the oil included within the pore-spaces of the reservoir-rock has been withdrawn from the well, connate water below rises to take its place. It is therefore a normal occurrence for oil-fields to produce large quantities of water during the later stages of depletion. In Figure 1 we show somewhat sketchily how these oil-reservoirs are situated with respect to structure.

When the Anticlinal Theory became generally accepted, an intensive and extensive campaign of surface-geology was undertaken which led to the discovery of many oil-reservoirs and reached a peak of activity and success in the early part of the third decade of this century. The utility of the methods of surface-geology, however, gradually lessened as the structures which have expressions on the surface were found. The problem of finding new oil-reservoirs became more acute and is, even today, one of the utmost importance. Whatever the market-conditions in the oil-industry happen to be, it must be remembered that in the final analysis, our entire civilization depends on its petroleum to an enormous extent and that the existence of known reserves is a fundamental factor in our national well-being. Then, too, so long as petroleum can be produced from the Earth it is questionable whether any "ersatzstoffe" can be produced economically.

The use of methods of geophysics in the search for geologic structures is of very recent origin--in fact, it is less than 20 years old. In this small interval of time the pace has been fast because of the competitive nature of the industry and, of course, because the prize

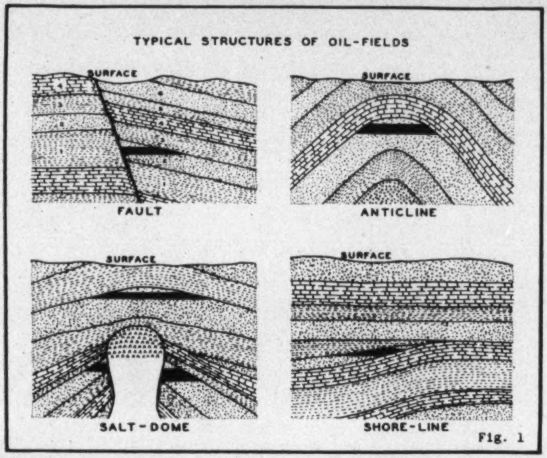


Fig. 1

sought is so high in value. For the same reasons, the progress in exploratory geophysics is unfortunately attended by claims of opportunists under various pseudo-scientific guises and, at times, by methods of thinking which are more wishful than rational. However, almost all other branches of activity have their analogies in "dowsers", "doodlebuggers", and similar phenomena of the human race.

At the present time the two outstanding methods used in petroleum-geophysics, which are the basic ones of the profession, are the gravimetric and seismic methods. A third method of importance is the magnetic method, in which the relative variations in intensity of the vertical component of the Earth's magnetic field are measured. The method is being used constantly and has the great advantage of economy. It is particularly successful in areas where key-beds of relatively high magnetic susceptibility exist.

Electrical methods of various types, too, have been used with some degree of success.

The gravimetric methods

If the various beds in a geologic basin are of different densities--as they usually are--then in the neighborhood of some structures with sufficient relief, the gravitational field at the surface reflects this situation. Depending on the distribution of densities, a structure may reflect itself either as a gravity "high" or a gravity "low", or embarrassingly enough, not at all. It can be readily shown--and practice has substantiated the matter--that for exploratory purposes an accuracy of about 0.0001 gal (1 gal = 1 cm/sec^2) is needed.

Probably the happiest "hunting ground" for the exploratory geophysicist has been the salt-dome area of the Gulf Coast of Texas and Louisiana. Here a shallow salt-dome will cause a gravitational "low" of the order of magnitude of three or four milligals, or, if it is covered by a heavy "cap-rock" of anhydrite, as a "high" of this magnitude. Some domes, of course, as has been indicated, show no gravitational anomalies.

Although in most of this paper the discussion centers around salt-domes, it should be understood that the ideas, with suitable changes, can be applied to "structure" in general.

Insofar as the use of the gravitational method for localized geological studies is concerned, its start should probably be dated sometime in the last few years of the nineteenth century, when Baron R. v. Eötvös built the first successful torsion-balance. This instrument was used with a great deal of success for geologic purposes in Hungary, but it was only in

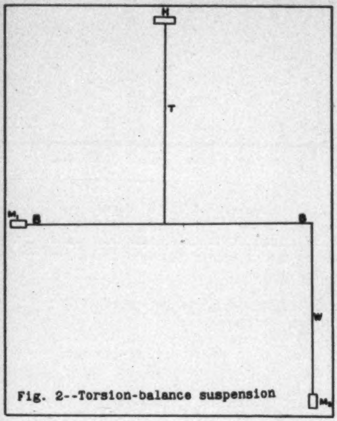


Fig. 2--Torsion-balance suspension

1922 that the petroleum-geologist seriously undertook the task of making use of it for his purposes. In that year the first survey in the United States was made over the Spindletop Dome, near Beaumont, Texas, in the Gulf Coast Area. The first geophysical prospect in this country was found by the torsion-balance in the Nash Area in Fort Bend County, Texas, and this is probably the first oil-deposit discovered by geophysical means in the world. The first well was drilled here into cap rock in 1924, and oil was found on the flank in 1926.

In principle the torsion-balance is essentially an adaptation of the Coulomb balance. In its usual form it consists of two systems of the type shown, the one having its lower weight on the one end and the other at the opposite end. These two systems are used for economy of necessary observation-time. Thus, in Figure 2, we have a beam BB suspended horizontally by a torsion-wire T from a "torsion-head" H. At one end of the beam a heavy mass M_1 is attached, and at the other a balancing mass M_2 is suspended at a lower level by a wire W. A Bamberg torsion-balance, ready for field work in a swamp, is shown in Figure 3.

It is a simple matter to write down the equations for the gravitational forces acting on the two masses M_1 and M_2 which produce a twist in the torsion-wire T. [Compare, for example, B. Gutenberg, Lehrbuch der Geophysik, pp. 517, and following.] It is this twist which is observed in each of several azimuths, with the result that we have, for the location of the instrument, two quantities. One is the horizontal gradient of gravity at the observation-station. The other quantity, called the differential curvature of the equipotential surfaces, is proportional to the difference of the principal normal curvatures of the equipotential surface at that point. The principal directions of the equipotential surface are likewise obtainable.

Fig. 3--Torsion-balance station in swamp

If we indicate the potential of a unit-mass at the observation-point due to the gravitational field by U, then, as a result of the torsion-balance measurements at a station, we find the values of

$$U_\Delta = (U_{yy} - U_{xx}),\ U_{xy},\ U_{xz},\ U_{yz},\ (\text{where } U_{xy} \equiv \frac{\partial^2 U}{\partial x \partial y}\text{, etc.})$$

at that station, in which the x-direction is eastward, the y-direction is northward and the z-direction is positively downward.

In practice, a station is made very carefully--about four a day can be made with one instrument--and the results are plotted on a map in the following manner:

The value of the gradient is plotted as the vector sum of U_{xz} and U_{yz}, the scale usually being 1 mm = 1 E, in which E, the Eötvös unit, is equal to 1×10^{-9} cm/sec²/cm.

The value of the "differential curvature" is

$$R = \left| g\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) \right| = \sqrt{U_\Delta^2 + U_{xy}^2}$$

Here ρ_1 and ρ_2 are the radii of principal normal curvature. This is plotted on the same linear scale, symmetrically about the observation-point, and in the direction of minimum normal curvature, which is obtained from the rela-

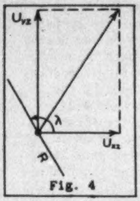

U_{yz}
λ
U_{xz}
R

Fig. 4

tions

$$\sin 2\lambda = (2U_{xy}/R)$$

and

$$\cos 2\lambda = (-U_{\Delta}/R)$$

Here λ (see Fig. 4) is the angle between the x-direction and that of the minimum normal curvature. The curvature is indicated by a line-segment, bisected at the station-point, whose total length is R and in the direction indicated by the value of λ.

At a torsion-balance station the local terrain may have a large effect on the data. Consequently, at each station an elaborate survey of the area in a 20-meter or larger radius is made and the "terrain-effect" is computed for the necessary corrections. This is a matter of converting the result of the proper triple integrals, expressing the effect of the soil above the base-plane, into forms which can be used quickly in a routine manner.

The mathematics of the measurements is a simple matter, but the applications to routine work form a veritable paradise for those who like to construct nomograms.

Stations are made at regular intervals, which may be from 200 to 500 or more meters long, depending on the detail desired.

The next stage in the process, after these "gradients" are plotted on the map, is to ob-

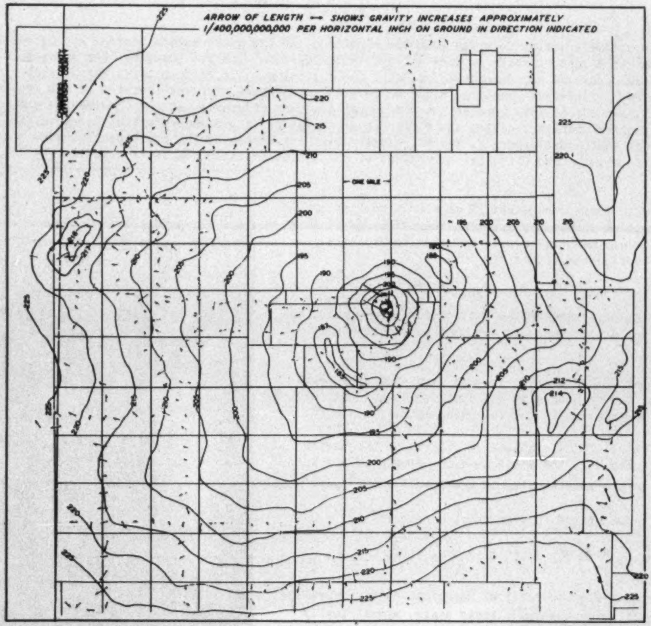

ARROW OF LENGTH ⟶ SHOWS GRAVITY INCREASES APPROXIMATELY 1/400,000,000,000 PER HORIZONTAL INCH ON GROUND IN DIRECTION INDICATED
CHAMBERS COUNTY
JEFFERSON COUNTY
ONE MILE
225
220
215
210
205
200
195
190
185
212
214
225

Fig. 5--Gradient-arrow and isogam, from torsion-balance survey, Big Hill Salt Dome, Texas tain the "iso-gravity" lines, commonly called isogams. This is done, of course, by one method or another of numerical line-integrations. As is to be expected, the methods of doing this expeditiously have been as ingenious as varied, but they all have as a goal the cutting down of the time necessary for completing the picture.

In Figure 5, we have a torsion-balance survey of the Big Hill Dome in Texas, with the isogams clearly indicating a gravity-maximum in the center. This dome is 800 feet deep, the top being covered with an anhydrite cap 500 feet thick. The center maximum is due to this heavy cap, but the "negative" effect of the main bulk of the salt shows clearly as the surrounding "trough".

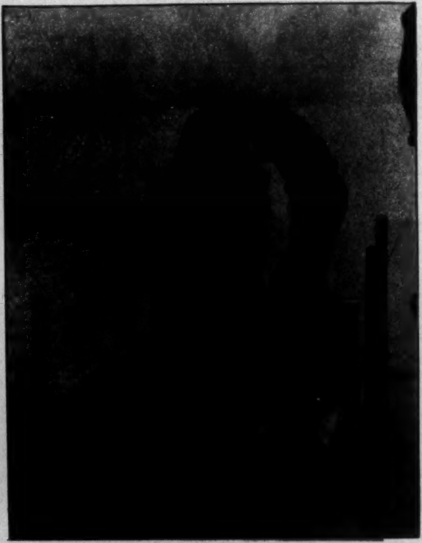


Fig. 6--Gravimeter in operation over open water

In recent years, the torsion-balance has gradually been displaced by the gravimeter. The development of the various types of these meters is a triumph of exploratory geophysics and instrumental technique, of which the oil-industry is justly proud. There are various types of such instruments in use, and each type depends on some mechanical means of counteracting the gravitational pull on a mass. The advantages of gravimeters are that they yield the relative values of gravity directly, that they are not sensitive to the local irregularities of terrain, that they do the work rapidly. Of course, elevation and topography have their effects, for which corrections must be made. Thus accurate topographic surveys are necessary. Whereas a torsion-balance could, in good country, make four stations in a 24-hour day (using photographic recording and night, as well as day, operation), a gravimeter can, in similar country, complete ten or more stations in a six- to eight-hour working day. In fact, the problem of surveying fast enough becomes a difficult one. In Figure 6 we show a typical gravimeter set-up in a bay.

For a while, too, some of the oil-companies were working on pendulums, but, although some excellent ones were developed, the gravimeters rapidly displaced them.

In making the gravity-map we have the problem of adjustment of error of closure whenever a "loop" is completed, precisely as it arises in topographic surveying. Here, then, as in many other phases of exploratory geophysics, we need the mathematics of "least-square adjustments", "probable errors", and other similar topics.

Once the gravity-map is obtained--corrected for elevation, topography, and latitude, with loops closed--it naturally becomes important to evaluate it in terms of subsurface-structure. The mathematicians will appreciate the difficulty of this problem. The "regional effect" due, most often, to basement-conditions must be eliminated, if possible. Some assumptions concerning depths and density-differences should be made, and very often can be made from data obtainable from geologic sources and well-samples. Here, too, the methods for such numerical interpretations are as varied as the number of people working on the problem. Often some success attends these solutions when checked by the drill; but it is felt, in general, that the gravimetric method is a powerful one in "pointing the way"--in isolating areas on which further work should be done--rather than a definitive one in the sense of making a definite location for a first well.

The seismic methods

The most definitive, as well as the most spectacular, methods used in the search for potential oil-structure are the seismic methods. It is generally conceded that to L. Mintrop

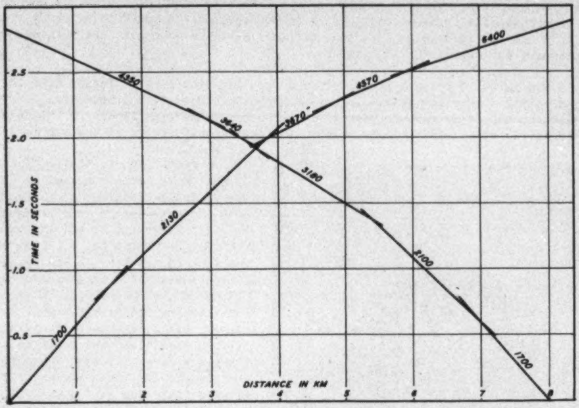
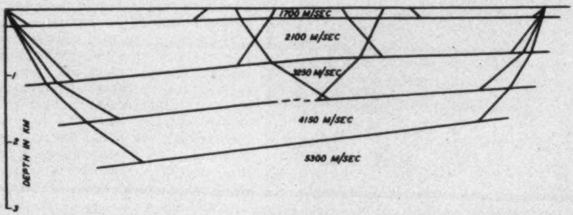


Fig. 7--Typical refraction-profile, time-distance curve, and interpretation



belongs the credit of realizing that artificial earthquakes could be used for geological exploration work and, in particular, for searching for potential oil-structures. Historically the use of blasts for seismic studies is much older.

From those first days in the early 1920's, with their mechanical seismographs and relatively crude refraction-methods and instruments, to the present in which technique, instrumental development, and field-organization have reached a high point in reflection-methods, the pace has been fast and unrelenting but preeminently successful in attaining the objectives.

Until about 1929 the method in general use was what we now term the refraction-seismograph method. At that time the reflection-seismograph technique was introduced and is now universally used, except for some special assignments for which the older refraction-methods are better adapted.

In its usual form the refraction-method embodies a shot-point and seismograph-stations in line with it spaced about 200 to 500 or more meters apart, the last being eight or ten kilometers from the shot-point. By radio the instant of explosion is transmitted to the recording stations. That instant and the response of the seismographs to the explosion are photographically recorded on moving sensitized paper, on which a 100-cycle tuning-fork leaves its traces. In that way the time of travel from the shot-point to each of the seismographs is obtainable.

In principle a seismograph is a mass suspended by a spring in a case and usually damped. Actually, the relative motion of the mass with respect to the case--which it may be assumed is fastened rigidly to the Earth--is, in present-day practice, converted in one of several possible manners into electrical energy. This, in turn, is sent through suitably designed amplifiers,

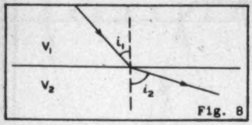


Fig. 8

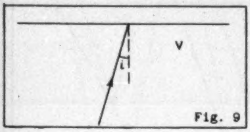


Fig. 9

and the motion is recorded photographically in an oscillograph. In Figure 7, a typical refraction time versus distance graph with the interpretation below is shown. It will be noted that, in refraction-work, only the first disturbance on the seismogram is used, in general. Sometimes, further "events" are noted--usually called "secondaries"--which may be used.

In the interpretation, Fermat's Principle and its corollary, Snell's Law, play the fundamental rôle. Stated roughly, but sufficiently accurately for our purpose, it is that a seismic disturbance, being wave-motion, travels along a minimum-time path. Fortunately, it is almost universally true that in the Earth's crust, seismic velocities increase with depth; otherwise, of course, refraction-exploration would be impossible. In its usual form, Snell's Law is

$$(\sin i_1/\sin i_2) = (v_1/v_2)$$

in which v_1 and v_2 are the seismic velocities in the two beds and the angles i_1 and i_2 are as shown in Figure 8.

In addition, we have what might be called the Law of Emergence-Angle; namely, that at the surface

$$\sin i = v(dt/ds)$$

in which $t = t(s)$ is the equation of the time-distance curve. Here, v is the velocity under the surface and i is the emergence-angle of the ray, as shown in Figure 9. The reciprocal of (dt/ds) is referred to as the apparent velocity of the arriving wave.

These simple considerations lead to some very interesting problems and to a host of consequent nomograms. In particular, when the velocity is apparently a "piece-wise" continuously increasing function of depth, some very interesting results are obtained. An example of such a situation is shown later. It should perhaps be added that, although any particular refraction time-distance graph has no unique solution, a set of adjacent graphs in an area tends toward uniqueness by demands of "reasonable" continuity.

A variation of the refraction-method is the technique of "fanning", peculiarly adapted for locating shallow salt-domes. So effective was it, in fact, that it practically scoured the Texas-Louisiana Gulf Coast of the shallow domes which were unsuspected until then. In brief, the method consists, as shown in Figure 10, of a shot-point and a number of seismographs arranged "fan-like" about it at distances of six to ten kilometers or so. If, in the area so covered, a salt-dome exists at a depth which may be about 1,000 or 1,500 meters, the travel-times of the waves to some of the seismographs will be shorter than are to be expected normally. The speed of seismic waves in salt is 5,000 meters per second or thereabouts, as against a speed of 2,500 or 3,000 meters per second in the sediments around it. The utility of the method is limited by the fact that the deeper domes are surrounded by sediments in which the seismic velocity is very nearly that in salt. The sketch in Figure 11 will serve to show the underlying ideas in this method of exploration.

The reflection-method

Some ten years ago the reflection-method of seismic exploration was introduced and this method has been attended by swift development and remarkable successes. Here, instead of using only the first disturbance on the seismic record, the arrivals of reflected energy from the various strata are studied.

In present-day practice, the method usually involves placing five or six seismographs on each side of, and in line with, a shot-hole. The seismographs are equally spaced with the outermost about 1,000 or 1,200 feet from the shot-hole. At that distance, too, another shot-hole is placed, so that by "leap-frogging" from shot-hole to shot-hole, the line is covered continuously. In Figure 12 this usual arrangement is sketchily shown. In Figure 13 we have three records obtained in this "continuous-profile" manner, with the reflections and instants of shot marked. The time-lines are photographically recorded from a tuning fork and are 0.01 second apart.

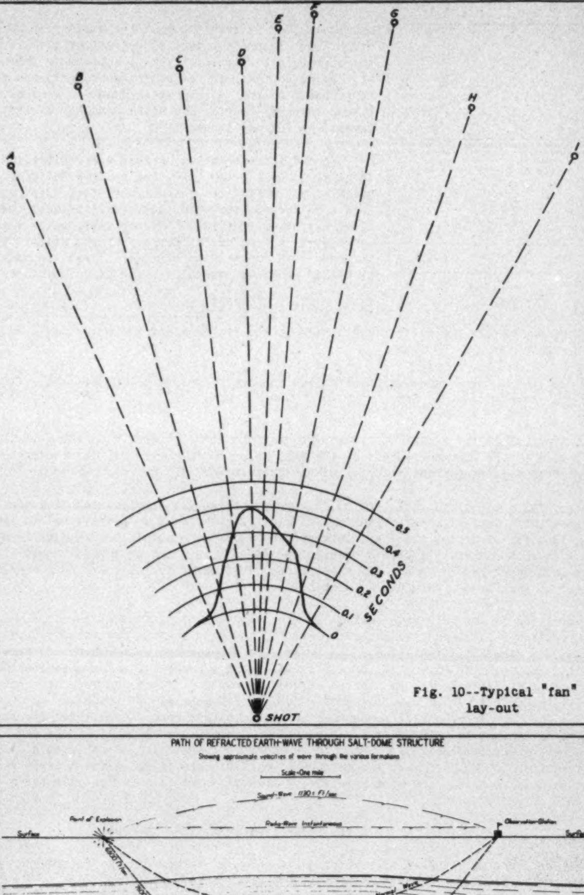
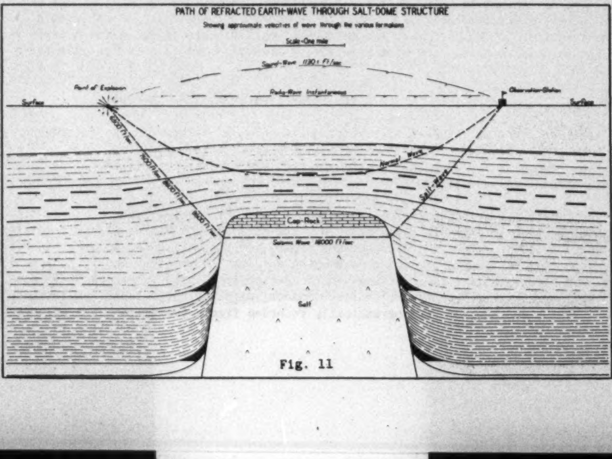

A
B
C
D
E
F
G
H
0.5
0.4
0.3
0.2
0.1
0
SECONDS
SHOT

Fig. 10--Typical "fan" lay-out



PATH OF REFRACTED EARTH-WAVE THROUGH SALT-DOME STRUCTURE

Showing approximate velocities of wave through the various formations

Scale-One mile
Sound-Wave 1130± Ft/sec
Point of Explosion
Radio-Wave Instantaneous
Observation-Station
Surface
Surface
6000 Ft/sec
7000 Ft/sec
8000 Ft/sec
9000 Ft/sec
Normal Wave
Salt-Wave
Cap-Rock
Seismic Wave 18000 Ft/sec
Salt

Fig. 11

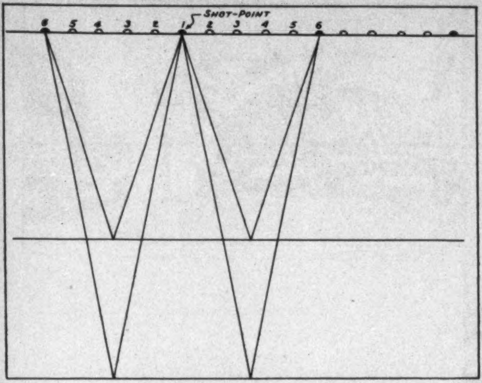


Fig. 12--Schematic view of continuous profiling in reflection-survey

As can readily be appreciated, the equipment is highly developed, very sensitive, but rugged; and the field-technique in obtaining these records is an impressive job of coordination and cooperation.

The shot-holes are drilled to a suitable depth, usually about 50 feet and the charge of dynamite necessary varies from a pound or so to about 25 pounds or more. This is in contrast to the hundreds of pounds necessary for a long refraction-line. The charge is tamped to the bottom of the hole, and the hole is then filled with water. This serves to minimize the loss of energy up the hole.

A few views of some of the field-operations in seismograph-exploration are shown in Figures 14 through 19.

To return to the discussion of the reflection-records shown in Figure 13, it will be noted that the reflection arrival-times were read at a certain phase--peaks or troughs--rather than at the onset, and that the correlations were made by having a common "channel" on adjacent set-ups. The onsets of the reflections can very seldom be determined and are really not important for the objective. The "refraction"-arrivals, commonly called "first-kicks", can be seen clearly, and these with the "up-hole" time serve to yield data necessary for correcting

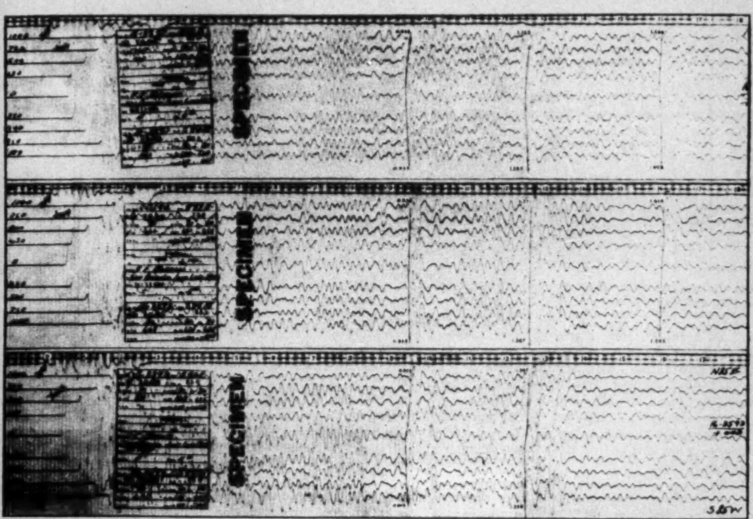


Fig. 13--Three "adjacent" reflection-records (arrows point to instants of explosions)

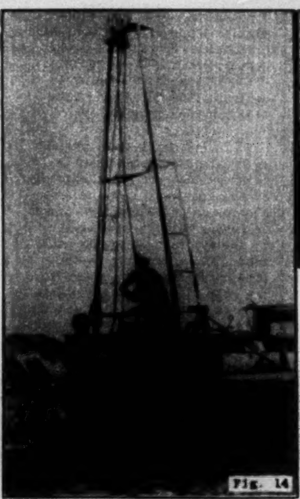


Fig. 14

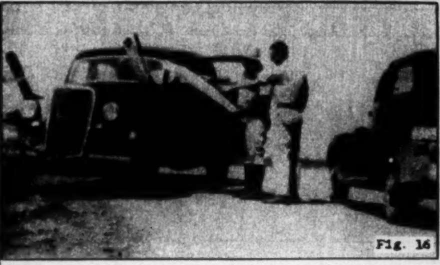


Fig. 16

Figs. 14-16
Drilling rig for shot-holes (14); operating truck (15); engineer and party chief (16)

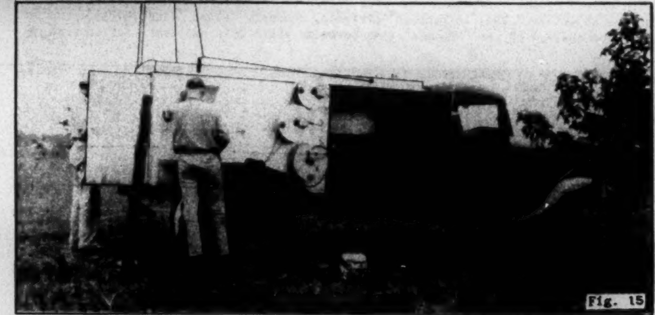


Fig. 15

the reflection arrival-times for the low-velocity layer, which is roughly comparable to the weathered layer.

On a reflection-record we have, then, the travel-times of reflections at each of five or six stations arranged along a line on each side of the shot-hole. In work which is not "continuous profiling", but rather "spotted", correlations are made by the "character" of the reflections; and the "step-out" or Δt between the first and last string, as well as the travel-times, are used to determine depths and dips. Of course, the complete subsurface picture is a three-dimensional one, but it is customary to simplify it to a two-dimensional one. This comes from the fact that we are interested more in the order of magnitude of the relief of the subsurface structure and the order of magnitude of depth rather than other quantities of highly exact accuracy.

Although there are in a single record ample data for computing depths, dips, and velocities of the various strata from a theoretical point of view, practically the accuracy (0.001

second) is such that the computations will be subject to large errors. As a matter of fact, the velocities used for these computations are not obtained from these data, but are rather obtained independently by other methods. Chief among the latter are the use of refraction-data and "shooting" of wells. In the first of these methods, the older refraction-results are used, in which velocities are computed as well as depths. Such data, however, are usually limited to less than 10,000 or 12,000 feet of depth, and they, at times, indicate embarrassing anisotropic effects.

In well-shooting, a seismograph is lowered by steps--perhaps 200 feet at a time--into a well, and the travel-times from shots on the surface to the seismograph are obtained. Such data have now been compiled over large areas in oil-producing regions with many interested companies cooperating in the ventures. Here, too, the depth of information is limited by that of the well, but the results are more convincing. The art of extrapolating these velocity-determinations to greater depths is one based on geologic knowledge and experience. At present, reflections from 20,000 feet of depth or more are quite common.

The study of velocity-depth data in their relation to geology is a very interesting one, which might very well be pursued with vigor. Some broad results become evident even without detailed study, and more extensive studies might yield some very valuable geological information. For the most part, average velocities are usually assumed by seismic computers, who then base their interpretations on the basis of rectilinear paths.

It may be pertinent to show how some velocity-depth relations lead, at times, to very interesting results and very interesting mathematical problems. For example, in the ideal case of n strata separated by horizontal planes in which the velocities are respectively v_1, v_2, ..., v_n, the travel-time of the reflection from the bottom of the nth stratum to a distance x from the shot-point is given parametrically by the equations

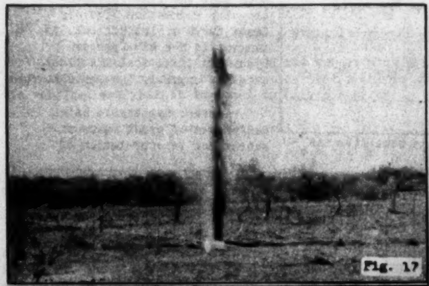


Fig. 17

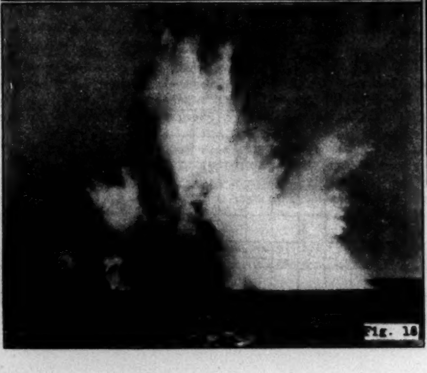


Fig. 18

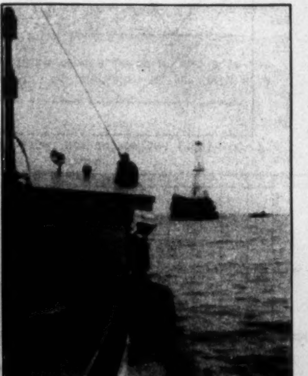


Fig. 19

Figs. 17-19
Reflection-shot immediately after explosion (17); refraction-shot in water (18); drilling rig in the Gulf of Mexico (19)

$$\left.\begin{aligned} x &= 2\sum_1^n \left(h_i p v_i/\sqrt{1 - p^2 v_i^2}\right) \\ t &= 2\sum_1^n \left(h_i/v_i\sqrt{1 - p^2 v_i^2}\right) \end{aligned}\right\} 0 \leq p < (1/\max v_i)$$

This curve is "hyperbolic" in character, having a horizontal tangent at $x = 0$. Its asymptote is the line (assuming that $v_{i+1} > v_i$), which is the refraction time-distance curve of the bed below the reflecting plane; that is

$$v_{n+1}t - x = 2v_{n+1}\sum_1^n \left[h_i\sqrt{1 - \left(\frac{v_i}{v_{n+1}}\right)^2}/v_i\right]$$

The value of the parameter p is precisely equal to the derivative, (dt/dx), of the curve at the point corresponding to that value of p. At the same time pv_i is the sine of the angle of the path with the vertical. This situation for three strata is shown in Figure 20. Note that the line representing the refraction time-distance curve of the kth bed, is tangent to the kth "pseudo-hyperbola", and that the first curve is actually a hyperbola.

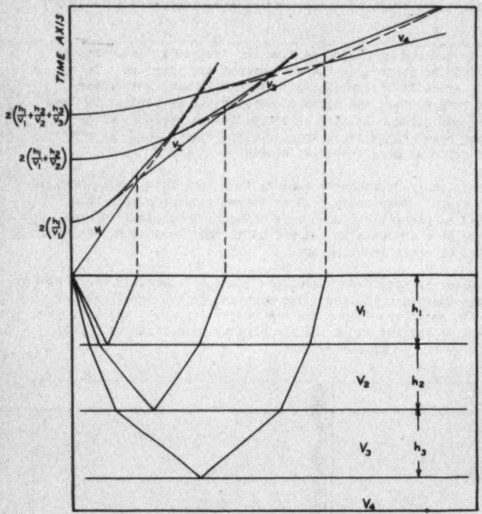

TIME AXIS
$2\left(\frac{h_1}{v_1}+\frac{h_2}{v_2}+\frac{h_3}{v_3}\right)$
$2\left(\frac{h_1}{v_1}+\frac{h_2}{v_2}\right)$
$2\left(\frac{h_1}{v_1}\right)$
v_1
v_2
v_3
v_4
V_1
h_1
V_2
h_2
V_3
h_3
V_4

Fig. 20--Ideal reflection and refraction time-distance curves

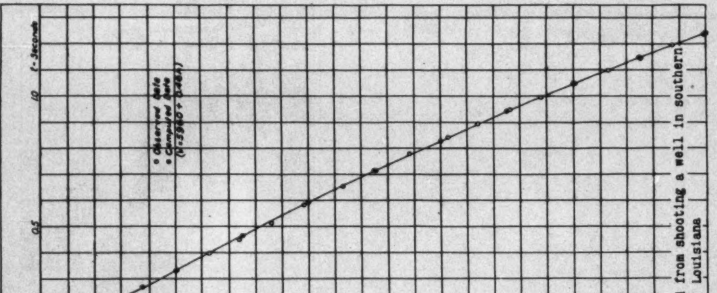


Fig. 21--Data obtained from shooting a well in southern Louisiana

Another hypothesis which has proven of great value in subsurface interpretation of reflection seismograph-work is that of a linear increase of velocity with depth.

In Figure 21 there is a time-depth plot of observed results in the shooting of a well in southern Louisiana. These observed points are plotted as circles. It will be noted that the plot is empirically smooth enough to suggest that there is a continuous increase of velocity with depth, rather than discontinuities at various depths in "stratified"-subsurface areas, as, for example, in west Texas. Of course, if the velocity increased continuously with depth, there would be no reflections. That we do get reflections is explained by the fact that there exist lenses of sediments, in the seismic sense, with physical dimensions of sufficient magnitude to act as reflectors, but which do not appreciably change the time-depth curves.

In considering various types of functions expressing the velocity as a continuous function of depth, $v = v(h)$, the simplest is probably that of assuming a linear increase. Such a hypothesis seems to be remarkably good over large areas. The mathematics is quite "elegant"--certainly so from the point of view of the pure mathematician, who will find a vast number of related problems arising from these considerations. If we assume a linear increase of velocity with depth, that is

$$v = v_0 + ah$$

the wave-paths will be arcs of circles through the shot-point, the centers of which lie in the horizontal plane which is (v_0/a) above the surface. The wave-fronts are the orthogonal trajectories of these paths and are spherical surfaces whose centers lie along the vertical through the shot-point.

By starting with the preceding time-distance-curve equations for n discrete strata, shown above, and passing to the limit in the usual manner of the integral calculus, we find that, if the velocity is assumed to be a function of depth satisfying relatively simple and "geologically reasonable" conditions, the time-distance curve for the reflections from a depth H is

$$x = 2 \int_0^H \frac{pv(h)dh}{\sqrt{1 - p^2v^2(h)}} \qquad t = 2 \int_0^H \frac{dh}{v(h)\sqrt{1 - p^2v^2(h)}}$$

Here $v(h)$ is the function expressing the velocity as a function of the depth h, and p is the parameter which exhibits the properties that $pv(h)$ is the sine of the angle of the path with the vertical and that it is equal to $[1/v(o)](dt/dx)$.

In particular, let us assume that

$$v = v(h) = v_0 + ah$$

Then, if the shot-point on the surface is at a distance d from a well, the travel-time to a seismograph at a depth h in the well can be shown to be

$$t = \frac{1}{a} \cosh^{-1} \left\{ 1 + \frac{a^2(d^2 + h^2)}{2v_0(v_0 + ah)} \right\}$$

Assuming v = 5,960 feet/sec and a = 0.46 foot/sec/foot, this curve would be that shown in Figure 21, the solid points being computed on this basis. One or two of the observed points are off 0.005 second, but all the rest are within 0.002 second--and this to a depth of 10,000 feet!

It has been our purpose to describe some of the methods and to indicate some of the problems of the petroleum exploratory geophysicist. That results are obtained is, of course, well-established. The final check on the existence of a structure is the drill. If the structure outlined by the geophysicist is not found, a review of the records to find out where the error exists, if possible, very often leads to new interpretation-technique.

From the mathematical viewpoint, the geophysicist must borrow from all the branches. He is primarily interested in applying--usually numerically--the results of practically everything in the mathematical kit to his problems of instrumental design and interpretation. Nonetheless, there exist, as by-products, a host of problems which, for the time, are academic in nature but are begging for investigation.

In conclusion, a note of acknowledgment to the host of friends who have helped in the preparation of this paper by their suggestions and criticisms is gladly added.

Humble Oil and Refining Company,
Houston, Texas

MATHEMATICAL PROBLEMS IN SEISMOLOGY

Archie Blake

Introduction

The purpose of this paper is twofold: (1) to call the attention of seismologists to certain mathematical methods available for application in seismology; (2) to bring to the attention of mathematicians a number of unsolved or partially solved seismological problems whose interest is largely mathematical.

The principal problems in the theory of seismic waves relate to the complexities due to inhomogeneities in the media and other causes, and to new methods available for the study of the interior of the Earth.

The problems of instrumental seismology are concerned with the extension and application of the theory of instruments already in existence and the development of the theory of new types of instruments--notably the strain-meter and the rotation seismograph.

The principal mathematical problems in the study of the destructive motion occurring in the epicentral region of a strong earthquake relate to the complexities encountered in determining the response of engineering structures to the motion.

The study of statistical seismology, and especially the periodicity-problem, indicates a need for new distribution-functions for the detection of effects which are not brought out by classical methods.

The complexities encountered in all these seismological problems emphasize the need for relegating to machines large parts of the calculations which are now performed by hand. To certain types of problem mechanical models, including various kinds of differential analyzer, appear well suited; to others the various types of counting device, and especially the machines which operate by punched cards, or punched or photographic tape.

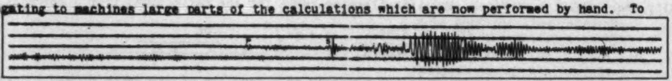


Fig. 1--Typical seismogram, showing condensational earthquake-waves, P, distortional waves, S, and surface-waves of larger amplitude, superimposed on weak background of microseisms

1. Seismic waves

The study of the various types of earthquake-waves has always been an engrossing part of mathematical seismology. The problems of greatest current interest are concerned with extensions to more complex problems than those dealt with in the classical theories--to inhomogeneous and otherwise anisotropic media, to motions involving strains which are too large to be treated as infinitesimal, and to the generation and propagation of the several types of surface-waves. The application of these results provides information about the deep interior of the Earth which is obtainable in no other way, and is capable of giving considerably more information than has yet been obtained.

1.1. The classical seismic waves--Figure 1 shows a typical seismogram, on which the first two waves to arrive, denoted by "P" and "S", involve respectively condensation-rarefactional and distortional motion. In an isotropic medium free from body forces, the velocities of waves of these two types, which are propagated independently, are respectively $\sqrt{(\lambda + 2\mu)/\rho}$ and $\sqrt{\mu/\rho}$, λ and μ being Lamé's elastic constants and ρ the density.

The seismogram shows also trains of waves of slower speed and larger amplitude than P and S. These long waves, "L", are surface-waves, in contrast to P and S which penetrate the body of the Earth. The first type of seismic surface-waves to be studied were those discussed by Rayleigh [see 12 of "References" at end of paper] which exist at the surface of a homogeneous elastic half-space, and in which the earth-particle vibrates (in a retrograde direction) in a vertical ellipse in the plane of propagation. Next investigated after the Rayleigh waves were the surface shear-waves of Love [13], which travel faster and have longer periods than the Rayleigh waves. Love waves are propagated in a layered crust; in them the direction of vibration of the earth-particle is horizontal and perpendicular to the direction of propagation. Observation does not agree with the elementary theory so well in the case of these two types of surface-waves as in the case of P and S, doubtless largely because of the complexity of the surface layers.

1.2. *Other types of seismic waves*--Every departure from the assumptions on which the classical theory is derived gives rise to complications in the types of elastic waves. Many of these have already been studied; others remain to the future. Some appear likely to involve only slight modifications of the elementary theory; others offer promise of throwing light on problems of considerable geophysical interest.

The classical theory of P- and S-waves supposes an isotropic (and therefore homogeneous) elastic medium admitting a strain-energy function and not subject to body forces, in which infinitesimal strains occur. The modification due to admitting gravitational forces was considered by Love [13], who found that the P-waves, but not the S-waves, are affected by gravity, but that the effects to be expect are small.

With regard to the strain-energy function, whose coefficients are the elasticities, seismology is fortunate in that the low thermal conductivity of the materials in the Earth and the short periods of earthquake-waves render the motion very closely adiabatic. This is not always true of the small specimens tested in the laboratory (thus heat conduction accounts for a large part of the damping in the vibration of reeds, as Zener [14] has shown); this fact accounts for part of the discrepancies between seismic and laboratory measurements of elastic constants.

The greatest part of these discrepancies, which are very considerable at low pressures, has been traced by Birch and Bancroft [15, 16], Zisman [17], and others [references in 18] to inhomogeneities in the materials. The ability of porosities and other inhomogeneities to affect seismic motion depends on the relation between the length of the waves and the scale of the inhomogeneities, which may be expressed by means of triple Fourier series, if only in a statistical sense.

It is probable that a large part of the advance of the theory of seismic waves in the next few decades will be concerned with the effects of various types of inhomogeneities. Jeans [19] has shown that two types of elastic motion are possible in a heterogeneous as well as in a homogeneous medium. Havelock [20] has studied the effects of dispersion. Arakawa [21] has investigated the magnitudes of the gradients of elastic constants required to modify the classical motions appreciably--and has found the gradients in the upper 20 km of the Earth's crust to be ample for this purpose. (The same is also doubtless true at other transition-zones within the Earth.) This matter has been studied also by Sezawa and Kanai [22]. The effect might be capable of yielding independent estimates of the density- and elasticity-gradients or their ratio.

When we confine ourselves no longer to anisotropies due to gradients in the properties of the medium but admit local anisotropies as well, the theory becomes much more complex. The strain-energy function has 21 independent coefficients and their gradients instead of only 2. It is mathematically a very attractive problem to investigate general types of seismic motion in such media, but the geophysical interest is confined chiefly to more special problems--for example, the stratification encountered in sedimentary beds or metamorphic rocks [Daley, 11, pp. 52 and 56]. The fact is, indeed, that present-day seismometric technique is barely adequate to determine the depths of discontinuities in homogeneous and isotropic structures; the development of the theory indicated above must go hand in hand with advance in other parts of seismology.

Seismic waves at any considerable distance from the earthquake-focus involve motions small enough to be appropriately treated as infinitesimal. The general case of finite strain demands a knowledge of the stress-strain relations and a theory of the propagation of large motion. The first of these two questions is a physical one: What is, in fact, the finite stress-strain relation for each material? Given the answer to this question, the derivation of the appropriate wave-theory is a mathematical problem. An important case has been considered by Birch [23, 24], using Murnaghan's theory of finite deformations [25], namely, the modification of seismic velocities (for infinitesimal strains) caused by the large initial strains due to the hydrostatic pressures deep in the Earth.

The treatment of the foregoing complexities in the theory of body waves remains largely to the future; the theory of surface-waves, on the contrary, has already become very extensive. The complexities are due to the many combinations of density, elasticities, thicknesses, etc., that may be assumed for the layers under consideration. In general, the mathematical problems here are all of essentially the same type, namely, to overcome the complexities in the application of known mechanical principles to the complicated structures. There are a few parts of the problem of surface-waves, notably those that depend on viscosity and other forms of internal friction, in which the physical laws require some further elucidation [15(p.128), 16, 26], but for the most part the difficulties are those of mathematical complexity. In general, with regard to the overcoming of this type of difficulty, there are two types of procedure that recommend themselves. One is to develop methods of approximation such as the well-known bounds and asymptotic formulas for characteristic numbers and functions, or the approximations, such as those of Rayleigh, illustrated by Jeffreys [27], in studying surface-waves. The other method is to develop machines by which forbidding complexities can be directly alleviated. This matter will be further considered in Section 5.

We cannot here attempt to survey the whole theory of seismic surface-waves, but some of the more pertinent contributions will be mentioned.

Uller [10] sought to unify the theory of various types of wave-motion and studied the generation of surface-waves as residuals in the reflection of body waves at the surface. Jeans [19] studied seismic waves in a gravitating sphere and found three new types of surface-waves, with the velocities of P- and S-waves at the surface. Arakawa [28] studied the influence of gravity on Love and Rayleigh waves (and found it to be small). Stoneley extended the theory of surface-waves to interfaces [29] within the Earth, and studied [30] Love waves in a triply layered medium, investigating the problem of inferring the thickness of the layers from the amplitudes of the waves. This investigation has been pursued by Sezawa and Kanai [31], who have also studied the dispersion of Rayleigh waves and have found two types of waves, with orbital motion in opposite directions and with different dispersion curves [32]. Sezawa finds also a possibility for anomalous dispersion of Rayleigh waves in a certain range of wave-length [33]. Honda [34], Arakawa [35], and Pekeris [36] have studied the effect of continuous variation in the density and the elastic constants (considered as functions of the depth) on Rayleigh waves. The studies of Mainka [37], Leet [38], and Macelwane [39] show quantitative discrepancy between the theory of Rayleigh and the waves which are commonly identified as Rayleigh waves on the seismograms. Macelwane [9, p. 123] considers the effect of inhomogeneity in the Earth's crust to be insufficient to account for the discrepancy, but the calculations of Pekeris [34] offer hope that the difficulty may be resolved. It would be desirable to pursue this problem further, and to extend Pekeris's theory to other types of variation than those represented by power-series.

There is need also for a theory of surface-waves in water-soaked terrain--waves intermediate in type between elastic and water waves.

For the most part the theory of seismic surface-waves is further advanced than observation. This is not due to lack of seismometric material, for the surface-waves are the ones that are recorded best by most seismographs. It is largely due to the fact that students of the theory of surface-waves seldom specify what one should look for in the trains of long waves appearing on seismograms. The customary reading of arrival-times of new pulses is rewarded in the case of the body-waves by the discovery of definite travel-time curves which are invariable from region to region to within a few seconds. But in the case of L-waves, especially those of Love type, the beginning of the motion is much less definite; one feels that the point one chooses as the beginning of the train of waves on the seismogram is subject to considerable arbitrariness, and that the measurement is accordingly not very valuable. The L-waves on a seismogram may last for an hour or two, with many regular trains of waves and perhaps a few rather clear beginnings of trains. Are these beginnings worth recording? Should the periods, amplitudes, and relative phases from component to component be recorded for the most prominent trains of L-waves? Is the variation of period of L-waves during the course of the earthquake worth noting? Would it be desirable to subject parts of the L-wave motion to Fourier or other type of periodic analysis? To what extent can the vibration characteristics and stratification of a region be deduced from the waves recorded there? (Cf. the work of Stoneley and others referred to above.) Of what significance are wave-to-wave comparisons between near-by stations? It will probably be necessary for theoreticians to specify rather definitely what they consider worth measuring in the L-group, for the task of reading seismograms is a burdensome one, and seismologists will not undertake new work in this direction unless it appears especially worth while.

There are several more problems of interest here. Seismograms are disturbed by an ever-present weak background of motion, consisting usually of regular trains of a few waves which die down to be replaced by new trains of similar nature. These motions are called "microseisms"; several types of them are recognized, varying in period from a few tenths of a second to about ten seconds. They appear to be of the nature of Rayleigh waves, and the causes of some types of them are believed to be known. Others, apparently caused in some way by atmospheric disturbances, are not well understood--a mechanism is needed to explain how they are set up [40]. (A biblio-

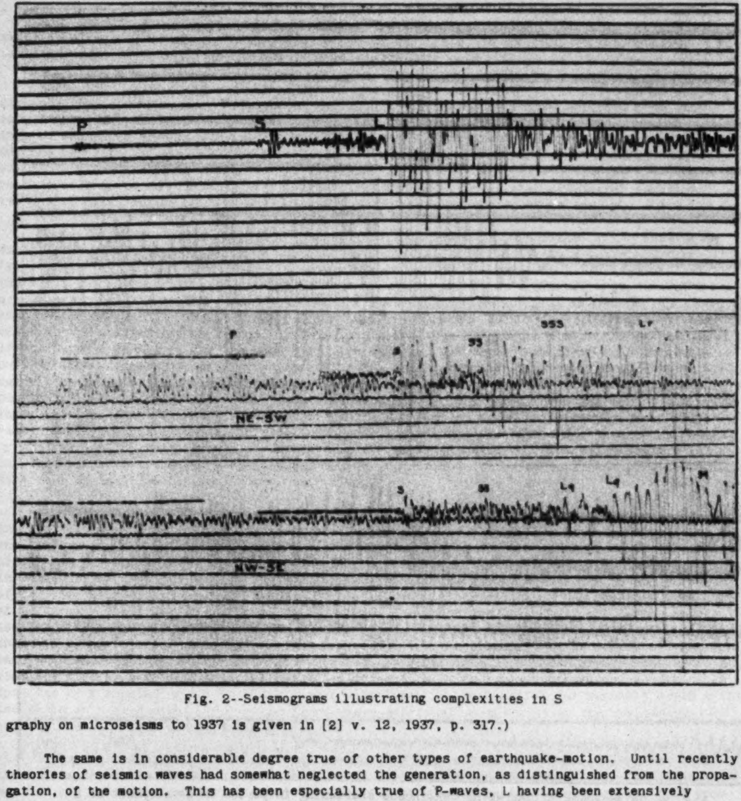


Fig. 2--Seismograms illustrating complexities in S

graphy on microseisms to 1937 is given in [2] v. 12, 1937, p. 317.)

The same is in considerable degree true of other types of earthquake-motion. Until recently theories of seismic waves had somewhat neglected the generation, as distinguished from the propagation, of the motion. This has been especially true of P-waves, L having been extensively studied, and S being presumed to arise from the shearing strain present before the earthquake occurs. (This explanation is now considered to apply to deep earthquakes as well as shallow, shearing strain being supposed to be capable of accumulating faster than isostatic flow can relieve it [11]; the source of the strain remains in question [41, 11, 7].)

Lamb [42] studied the type of motion to be expected in an isotropic medium from a singlet source, and Sezawa and Nishimura have extended the theory to doublets and multiplets. Important work on various aspects of this problem has been done by Inouye [44], Kanai, Nakano, Nishimura [45], Sakai, Slichter, Takayama [46], Uller [10], and others. The results of these studies lead to motions in some respects very different from what is observed on seismograms; in particular, the theories call for brief motions--a single pulse, or one or two waves--instead of the trains of more or less irregular waves that are observed. Lamb [42] supposed that the difference might be due to the existence of multiplicities in the original motion, but the subsequent improvement of seismographs has largely, though not entirely, negated this explanation. Jeffreys [7, p. 94] attributes the effect to reflections and to dispersion (and it seems probable that these are responsible for a considerable part of the phenomenon), but dismisses the existence of oscillators in the Earth's crust capable of producing the effect. But the stratification

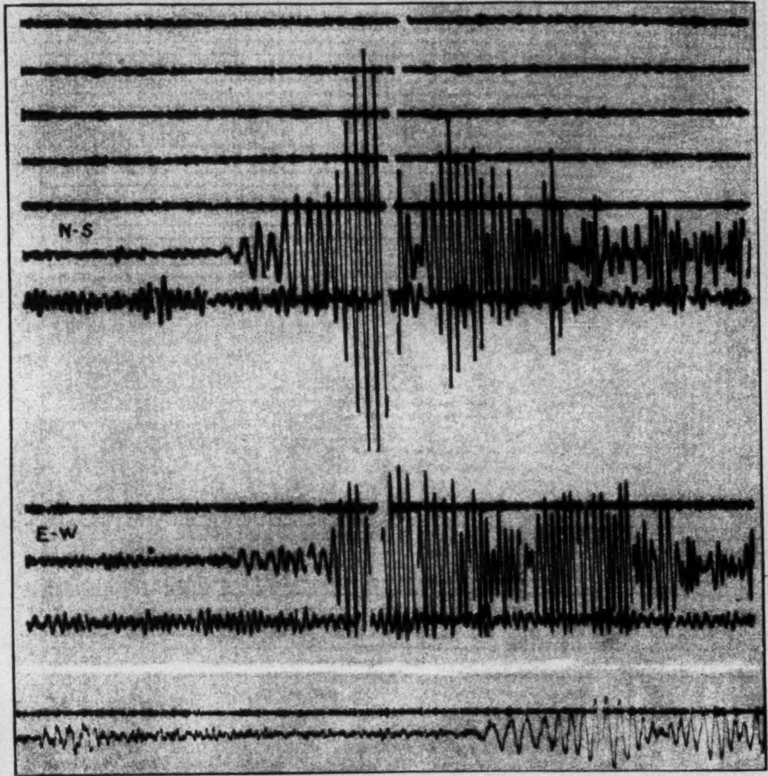


Fig. 3--Seismograms illustrating continuous shortening in period in development of L

in the Earth may at many places be sharp enough to provide layers which behave approximately as discrete oscillators. Indications of this kind for shallow layers have appeared in the work of Köhler and others. Slichter and Gavrilovich [47] find that many of the irregularities in the records of seismic waves may be due to reflections at the boundaries of the upper layers. Part of the effect may be attributed to the failure of Fresnel's principle in an inhomogeneous medium. The conversion of a sudden impulse into a train of waves involves a lengthening of the period; this might reasonably occur as the rays penetrate the graver strata, but this result is to be expected also from internal friction, as is indicated in the studies of Sezawa [48], Gutenberg [11, p. 367], and Jeffreys [7, p. 114]. But the laws of internal friction in solids are not in a satisfactory state [15, 16, 26]; this problem, like the calculation of elasticities to be discussed in Section 1.3, awaits the application of atomic theory.

Seismograms (see Fig. 2) show many anomalous motions which await explanation, such as a long low swell which sometimes accompanies P, or a strong wave, apparently of S-type, which often appears a few seconds after the "true" S, and is frequently read instead, or a continuous shortening of period which characterizes the development of the first L-waves (see Fig. 3).

1.3 Seismic evidence on the interior of the Earth--Most geophysical sources give merely average values for the density, rigidity, etc., of the Earth, in contrast to the detailed variation with depth which is obtainable from the seismic data.

Recording the times of arrival of the P- and S-waves at seismological stations at varying distances from the epicenter of a strong earthquake, combining data thus obtained from many earthquakes, and supplementing this information by amplitude measurements and by near-by recordings of the waves from timed explosions, one obtains travel-time curves from which the apparent surface velocity is calculated as a function of the distance. The apparent surface velocities of the two types of waves depend on the variation of their velocities with depth; the latter can be deduced from them. (This calculation involves the solution of Abel's integral equation; two fairly satisfactory ways of arranging the computation have been given [49, 50, 51].)

The velocities of both P and S increase with depth down to the surface of the core (at a depth of about 2,900 km). Thus the elasticities increase faster than the density and cause the trajectories to be considerably bent, concave upward. A seismic ray reaching a depth at which the speed increases very rapidly might be bent enough to emerge at the surface of the Earth at a smaller epicentral distance than a less deeply penetrating ray, and thus give a triple-valued travel-time curve. The calculations of Slichter [52] indicate that such cases probably occur often in the shallow layers reached in seismic prospecting. The variation in the deeper parts of the Earth's mantle is for the most part more equable, but there is a sharp bend in the P-curve in the neighborhood of 20°, which may be the intersection of two branches of a triple-valued curve (the later arrivals being harder to observe on account of the disturbance on the seismogram caused by the first arrival). The S-curve shows a similar sharp bend near the same place. The interpretation adopted, whether of sharply bent single-valued travel-time curves or of triple-valued curves, makes a considerable difference in the velocities deduced for the depths reached by the rays in the neighborhood of the bend in the travel-time curves, but not elsewhere [53]. In other respects the seismic velocities down to the surface of the core are not very critically dependent on small variations in the observations.

Recalling that the velocities of P and S are respectively $\sqrt{(\lambda + 2\mu)/\rho}$ and $\sqrt{\mu/\rho}$, we see that seismic measurements yield apparent adiabatic values of the ratios of these two elastic coefficients to the density. It is of interest to inquire whether the variations of the elasticities and the density can be obtained separately. The problem is made no more difficult by introducing the pressure, for it can be computed (apart from slight errors due to the strength of the rock, especially near the surface) from the density-distribution. As there is some hope of estimating the temperature from theories of the formation and history of the Earth (though the estimates thus obtained are usually higher than the Curie point at which materials lose their ferromagnetism), the equation of state, connecting the pressure, temperature, and density (and chemical composition) becomes available for the determination. The pressure-density relation for a fixed sample of material subjected adiabatically to various pressures is obtainable from the bulk modulus, $k = [\lambda + (2\mu/3)]$; but unfortunately the equation for variable depth within the Earth is affected not only by the temperature-distribution but also by the unknown variation of the chemical composition [24, p. 470].

Supposing temporarily, then, that we are dealing with a chemically homogeneous and isothermal layer within the Earth, or a layer whose composition and temperature vary in a known way, we can determine the variation of the density and elastic constants with depth if we know the equation of state applicable to the large strains due to the high pressures deep in the Earth. This problem has been considered by Birch [23, 24] on the basis of Murnaghan's theory of finite strain [25].

Two points of view have been prominent in the history of elasticity theory, namely, (1) that of Cauchy, who (in addition to providing a considerable body of correct theory) sought to compute elastic coefficients in terms of central forces between the molecules, and (2) the point of view of Green, who contented himself with the less specific but correct results obtainable by the application of conservation laws. Murnaghan's theory represents a further advance in the direction followed by Green. It possesses two fundamental advantages, its invariantive form and its exact treatment of the conservation of energy, which in former theories had been dealt with no more closely than was required for an infinitesimal elastic theory. But it is subject to the disadvantage of all methods which seek to make the conservation laws suffice in that it neglects the detailed properties of the materials which it seeks to describe. There are many equally justifiable alternatives to Murnaghan's power-series expansion of the potential-energy density in the traces of the strain tensor.

Atomic theory is now far enough advanced to permit a return to Cauchy's program of explicit treatment of elastic forces, with the essential difference that we are no longer confined to central forces, but to those provided by the quantum theory. Progress has been made in recent years in this direction by Landshoff [54], Mott and Jones [55], Seitz and Johnson [56], Teller [57], Bardeen [58], Jensen [59], Fermi, Shockley [60], and others. The formula to be used depends on the temperature-and pressure-range considered, and in some degree on the material. Not very much is known about the temperature inside the Earth; it may reach a few thousand degrees. The pressure is better known [61]; it reaches about 1.4×10^{12} dynes/cm^2 (1.4×10^6 atmospheres) at the surface of the core and about 3.5×10^{12} dynes/cm^2 (3.5×10^6 atmospheres) at the center of the Earth. These pressures are high when considered in terms of ordinary experience but not in comparison to the forces studied in atomic theory. They are probably sufficient to cause interpenetration of a few of the outer electronic shells of the atoms involved--that is, they are comparable in magnitude to ordinary chemical forces, a few volts.

In the lower pressure-regions--probably within the Bridgeman range (to about 5×10^{10} dynes/cm^2) and perhaps a little beyond--the methods of Landshoff [54], Seitz [56], and Bardeen [58] appear to be suitable. Bardeen gives a cubic expression for the energy of the alkali metals (which are simplest to treat) in terms of the linear compression. (This cubic relation is not merely empirical.) According to Seitz the same methods should be applicable to other materials than the alkali metals, at the cost of considerable increase in complexity in the mathematics.

At very high pressures the electronic shells will be crushed and the variation of the pressure will approach that of a degenerate electron gas, the pressure varying inversely as the 5/3 power of the volume. But according to Fermi the deviations of this description are still presumably about a factor of three at pressures in the neighborhood of 10^{14} to 10^{15} dynes/cm^2, which exceed the pressure at the center of the Earth by a factor of about 100.

The intermediate range, from about 10^{11} dynes/cm^2 to 3.5×10^{12} dynes/cm^2 at the center of the Earth, has not yet been very satisfactorily treated, but Jensen [59], using the Thomas-Fermi atom-model, has obtained curves for various elements which approach that for the electron-gas at very high pressures, though they are considerably in error in the experimental range. It seems probable that these formulas could be improved enough to make reasonably accurate pressure-density relations available for the whole range from the experimental pressures to the center of the Earth. It is not necessary to compute the coefficients ab initio; they can be evaluated in terms of observed compressibilities and other measurements, as by Bardeen. For this purpose it is of interest to note that laboratory technique is becoming capable of reaching considerably increased pressures. Pressures of 2×10^{11} dynes/cm^2, corresponding to depths of about 600 km, have recently been attained at the Geophysical Laboratory of the Carnegie Institution of Washington, and it is not too much to hope that these pressures will be improved by a factor of two or three in the not too distant future.

For the purpose of determining the density-variation within the Earth, however, a hard-won equation of state is of little avail if the problem is affected by the unknown variation of chemical composition. It does not appear intrinsically probable that the Earth should contain any very thick shells which are chemically homogeneous, or, indeed, that there should be anything analytic or very regular about the variation. There is, however, an important source of information which we have not yet mentioned, namely, in a liquid Earth the materials should arrange themselves in a state of stable equilibrium. (The condition is not precisely that the denser materials be at the greater depths--the bulk moduli and chemical affinities enter too.) If the Earth was at one time liquid, this state should have been approximately reached; even now the strength below 100 km or so does not appear sufficient to support large distortional stresses for a very long time. But some of the equilibrating processes, such as heat conduction and the decay of uranium, proceed so slowly that they will not have advanced very far even in the 2 or 3×10^9 years of the age of the Earth. Given sufficient knowledge of the chemical and physical properties of matter at high temperatures and pressures, and the initial state of the Earth, it would be possible to trace the whole of its history, rather than merely to describe its final equilibrium state. This, of course, has already been done in a crude way--the core of the Earth is supposed to be of nickel-iron, surrounded perhaps by layers composed successively of sulphides, etc. (?), dunite, basalt, and granite. In the present state of our knowledge these conclusions are very precarious. Rough estimates of this kind are perhaps all that will be possible for some time to come; but there seems to be hope, in view of the apparent relative scarcity of most of the elements, that the initial state could be well enough represented with comparatively few unknowns--the seismic data would be sufficient to render such a problem determinate. Here, as in the equation of state, the principal ingredient lacking is the application of quantum theory to the deduction of chemical properties at high temperatures and pressures.

The core of the Earth--below about 2900-km depth--is supposed to be liquid, as judged by the failure to detect distortional waves through it. There is still some room for doubt about this point. The effect could be due to the damping out of the S-waves by internal friction instead of lack of rigidity, though it would appear that such friction should also affect the condensational waves. It is also true that the seismically determined rigidity of the mantle is enough to satisfy the requirements of tidal theory for the whole Earth--there is not room for much rigidity in the core [11, p. 352]. But the rigidity effective in seismic motion of period only a few seconds may be an appreciably different property from that effective in resisting tidal forces. In general, on account of its greater inaccessibility, the core is less well understood than the mantle [62].

The foregoing discussion is confined to the results obtainable from the classical arrival-times of the P- and S-waves at the seismological stations. It remained for Slichter [63], Langer [64], and Pekeris [65] to call attention to a new and powerful method: if we measure not only the arrival-times but the form and size of the waves as well, then it is possible to deduce not merely the ratios (λ/ρ) and (μ/ρ), but all three of λ, μ, and ρ. That at least some progress can be made in this direction is very easily seen, for supposing that the velocity of sound in air were the same as in the Earth's crust, this fact would not inhibit the reflection of P-waves at the surface: the propagation of the motion depends, then, on all three quantities instead of only the ratios. Pekeris's treatment applies only to a horizontally layered half-space, and there are some questions of convergence which he has left unanswered. The settling of these questions and the extension of the results to the sphere present a very attractive problem.

It is especially desirable to solve this problem without use of power-series--not only because existence and unicity proofs for solutions of differential equations should generally be obtainable on weaker assumptions, but because the complexity of layering in the Earth offers but slender hope (despite Weierstrass's theorem on the approximation of continuous functions by polynomials) for the practical application of power-series over any considerable range of depth. Stevenson's [66] use of a discontinuous function as an example to test the convergence of the power-series in an analogous problem in electrical conduction is inappropriate, but his conclusion that methods of successive approximation should converge where many other methods fail is in accord with almost universal observation. Another possibility is that trigonometric series based on some weight function which decreases with the depth as in Laguerre polynomials (for example, $\exp(-1/r)$, etc.) might be appropriate to this problem.

2. Seismometry

The mathematical problems of instrumental seismology may be classified into two types. The first consists in working out the theory of instruments already in existence, the extension of this theory in various directions that suggest themselves, and the application of the results to the improvement of the instruments. The second type of problem consists in developing de novo a theory suitable for the design of various new types of instrument.

2.1. Teleseismic instruments--The effort to increase the sensitivity of seismographs designed to record distant earthquakes has led in two directions, namely, (1) to an increase in the delicacy with which the seismometer pendulum is suspended, together with shortening of the radius of oscillation to obtain increased static magnification, on the one hand, and (2) to galvanometric recording on the other. Each of these types of seismograph design leads to mathematical problems: The first in that types of motion not contemplated in the elementary theory of the instrument may become comparable in magnitude to the response intended to be measured, or even larger; the shaking-table tests of McComb and Wenner [67] show the presence of such effects in the response of the Wood-Anderson seismometer to short-period motion, and effects of similar nature would doubtless appear if other types of instrument were subjected to similar tests. In galvanometric instruments the fact that the galvanometer is not tightly coupled to the pendulum means that the number of degrees of freedom is increased and the order of the differential equation of motion is higher than that (two) of the mechanical instrument. The theory of the galvanometric seismograph has been studied by Galitzin [68], Wenner [69], Sohon [8, part II], Benioff [70], Blake [71], Grenet and Coulomb [72], Scherbatskoy and Neufeld [73], and others, but many interesting problems are yet to be discussed. For example, the discriminant of the characteristic equation of the ordinary seismograph vanishes in the important case of critical damping--what is the corresponding application of the two invariants of the fourth-order equation of a galvanometric seismograph? It is very useful in seismology to be able readily to trace the connection between the constants and the performance of the instruments.

For the purpose of measuring the times of arrival of seismic waves no very elaborate theory of the seismograph is required, but it has been seen in Section 1 that a number of important problems, such as the propagation of L-waves, the nature of the initial movement at the focus, and the application of the method of Pekeris to the determination of the variation of the density within the Earth, depend on the obtaining of quantitative information which present-day practice is not in a position to supply.

At some stations, Honolulu for example, the microseisms present a very acute problem, being often larger than the P-waves of important earthquakes. The short-period Benioff seismographs have been found so very successful in recording P-waves that it seems probable that such an instrument, with period well below that of the principal microseisms, but with low enough sensitivity not to be unstable at the site, might effect considerable improvement in the recording of P-phases at Honolulu.

These considerations illustrate a fact of much practical importance in seismometry. Every site has its own peculiarities which call for a special set of instrumental constants--period, magnification, and damping--best suited to that locality. The construction of many kinds of instrument, testing them at the sites where they are intended to be operated, and redesigning them if they appear unsatisfactory--all this contributes heavily to the cost of owning and operating a seismograph, and tends to keep down the number of good stations.

It should be possible to design a general-purpose seismograph which could be produced in quantity at small cost--an instrument with adjustable period, magnification, and damping, which could be adapted to almost any site. The instrument must be capable of yielding quantitative information on the amplitudes of the motions recorded. At an outlying observatory, if a galvanometer filament is broken and a new one installed, the constant of the new filament must be determined if the instrument is to yield quantitative results; this is often a matter of some practical difficulty in the field, but the measurement could be made by means of some such testing device as that of McComb and Nelson incorporated in the seismograph. Such a device is almost necessary if the constancy of the instrumental performance is to be depended on over any considerable time.

The instrument should be tilt-compensating (probably galvanometric) and should be equipped with some means for suppressing the violent response to near-by shocks by which most of the motion after the arrival of P is lost in strong earthquakes. (This could be accomplished, for example, by using in the magnification- or damping-circuit a vacuum-tube whose output would become non-linear for large motions; but the requirements of practical field-operation must be considered.) The vertical component should be stable and should have constants not very different from those of the horizontals.

The development of the theory of the seismograph toward the design of an instrument meeting these requirements is the principal mathematical problem of teleseismometry.

2.2. Strong-motion seismometry--The measurement of the motion of the ground or of engineering structures in the epicentral region of a strong earthquake is a quite different problem from that of classical teleseismometry. The determination of the absolute time of arrival of the new trains of waves is no longer important; what matters are the amplitudes, periods, and forms of the waves. It is necessary to use a much more open time-scale than in the ordinary seismograph. Various types of instrument, ranging in period from less than one-tenth of a second to ten seconds, have been constructed [74] or proposed [75] for this purpose, but this development has all occurred in very recent years, and the theory of these instruments has been little studied. The theory of the instrument is inseparably bound up with the purpose it is intended for, which will be discussed in Section 3.

2.3. Strain-meters and rotation seismographs--We may think of the motion of the ground as comprising three components of linear displacement, three components of rotation, and three components of strain. Almost all seismographs are designed to record the displacement without regard to the rotation or the strain, but these latter may be expected to assume increasing importance as seismology progresses. For example, there is evidence that much earthquake damage is due to the tearing apart of the foundations of buildings and other structures by the ground strain rather than to the effects of inertial forces. Benioff has constructed a strain-meter at the Seismological Laboratory at Pasadena and has published a brief discussion of the theory of the instrument and its advantages in teleseismic work [76]. A theoretical investigation of the capabilities of the various possible types of strain-meter--of volumetric, interferometric, and perhaps polarimetric instruments, for example--is much to be desired.

The rotation seismograph presents problems of special interest on account of the failure of finite rotations to add commutatively. The theory has remained untouched. The instrument may be of the inertial type, or may measure rotation relative to some given direction, such as the Earth's gravitational or magnetic field or the direction from the seismograph to some visual object. (A moving-picture camera would serve this purpose.) The only rotation seismographs thus far developed have been tilt-meters; these instruments are of special interest on account of the possibility that Earth-tilts may presage destructive earthquakes.

3. The strong-motion problem

Extensive study of seismology with a view to obtaining data of value in earthquake-resistant design has come into being only in very recent years [77, 78]. The problem is to measure those elements of the motion which may be pertinent in causing damage, to determine the response of various structures to such motions, and on the basis of these results to formulate rules for design.

Stated in this way the problem appears to be straightforward enough, at least in principle. The difficulties here as elsewhere arise on account of the complexity of the phenomena studied. The instrumental problem has been mentioned in Section 2.2. Supposing satisfactory measurements to be available, what problems remain?

The analysis of the response of a structure into characteristic or normal components satisfying linear equations depends on the treatment of the strain-energy function as quadratic. But the purpose of investigating the response to destructive earthquake motions requires consideration of strains much exceeding the limits within which Hooke's law remains valid; then the effects of the various components of ground motion can not be treated separately. The linear theory can be used as a first approximation, which will be accurate enough for some purposes and can serve as a start in the more complete solution of the problem by methods of successive approximation, variation of parameters, etc. The application of these methods to this problem has not been made.

Another complication arises from the presence of off-diagonal terms in the damping (Rayleigh dissipation-function) when the kinetic and potential energies are simultaneously reduced to sums of squares. Rayleigh [79] remarks that if the damping is small, the characteristic motions will be approximately the same as if there were no damping, so that in each mode the disturbing velocities due to the other modes will be small. Then, since all the damping coefficients are small, the disturbing terms will be of the second order, and the motion of each mode will follow approximately the ordinary seismograph-equation.

This case of small damping is the one most commonly encountered in buildings, bridges, tank-towers, etc., and the existence of normal modes of oscillation has been verified for them. The off-diagonal terms in the dissipation-function may possibly have considerable importance nevertheless. These terms are associated not only with damping but with interaction among the normal modes, by which any mode being supplied with too much energy might share its motion with the other modes and thereby escape destruction. Conditions would be especially favorable in the case of a dissipation-function with many considerable off-diagonal terms. A widespread maxim favors unity and simplicity in earthquake-resistant design. There is no question about the value of simplicity in the case of unstable chimneys and insecure ornamentations that may readily be shaken off a building, but the case is not so clear as regards the fundamental structure. It is of interest to recall a study made by Martel [77, chapter 8] on the effects of the Long Beach earthquake, in which it appeared that damage to buildings adjacent to others was less than to those that were isolated, and less for those with buildings adjacent on two sides than on one. This observation can be explained qualitatively either in terms of increased effective rigidity or interaction; the case for interaction is supported by Martel's observation that the percentage of damage at Long Beach decreased with the number of stories of the building, though this could be explained by the increased difference between the period of the building and of the earthquake-motion. A quantitative discussion is needed. It is probable that coupling of the different modes of oscillation should usually tend to protect a building or group of buildings rather than weaken it, provided the coupling is strong enough to carry the loads imposed upon it. But a building consisting of several wings weakly connected together, for example, would naturally tend to be ruptured at the junctions.

Another complication in the strong-motion problem relates to the yielding of the ground. One customarily thinks of the ground as imposing inexorable motion on the building, the effect of which may be treated in terms of forces of kinetic reaction. But it appears that in some cases (such as that of short-period vibrations acting on a rigid building in soft ground [80]), the ground may yield considerably. The normal modes of oscillation are altered in period and type [81, 77 (pp. 49-52)], a new opportunity is afforded for the building to lose energy, by radiating it into the ground (cf. Sezawa and Kanai, [78]), and the calculation of the motion is rendered more difficult.

If the yielding of the ground is small, the simplest method of calculating the response of the building may be to compute, first, an approximation to the building motion on the supposition that the ground does not yield; next, to calculate the yielding caused by this much building motion, and then to use the ground motion as thus modified to repeat the calculation until the result is closely enough approximated. But it appears that the analysis into building and ground is itself somewhat artificial; it may often be better to treat the two together as one complex oscillator, in which the foundation of the building merges insensibly into the ground.

The foregoing considerations indicate that a treatment of the response of structures to earthquake motion may become very complex. Direct methods for overcoming the complexities, such as the mechanical methods to be discussed in Section 5, are much to be desired, but it is also worth while to seek methods of simplification by which those complications which involve only small corrections may be avoided from the start. The disparity among the intensities of earthquakes is very great, and high accuracy in computations of building response, therefore, of little value. We mention several simplifying assumptions in the strong-motion problem, with a few remarks on each.

The assumption that the rotation and strain of the ground may be ignored is probably fairly true for the rotation but not for the strain. The inertial forces due to rotation are probably very small; gravitational forces due to tilts are measured alike by seismograph and building (insofar as the tilting of a seismograph represents that of the whole plot of ground on which a building stands) and may therefore be included with the translational motion. Ground strain appears to be important and requires to be measured and studied.

Non-linearity in the response of a structure is important but can probably be taken account of accurately enough by approximate methods. Its chief effect appears in the change in the periods of structures with amplitude. Small changes in period are important because of resonance, the damping of engineering structures usually being small. Theoretically the period of an oscillator might either increase or decrease with amplitude, but increase seems much more common. If various structures are consistent enough in this respect, it should be possible to formulate this regularity in a rule of sufficient validity for many purposes of engineering seismology. The results of the measurement of the vibration of buildings due to wind tremors and the action of small shaking-machines would then take on a new significance. It appears probable that measurement of microseisms and the simultaneous measurement of the response of buildings to them would be very desirable. It is especially worth while to measure the motion of the various parts of a building simultaneously.

The assumption that the various modes of oscillation of a building are uncoupled and that the building does not radiate energy into the ground leads to a very valuable analysis of the motion, the earthquake being represented by the family of response-curves of simple oscillators subjected to the ground motion considered as inexorable. One would not wish to forego this analysis; it is better to use it with its limitations in mind. Methods of calculation already mentioned may be used to improve the approximation.

4. Statistical seismology

All branches of science have become increasingly conscious of the statistical aspects of their problems. A number of interesting questions arise in seismology, some of which are important throughout statistical theory.

4.1 The reduction of seismogram-readings and the computation of travel-time curves--Two methods have been used extensively in the revision of seismological tables by use of new readings. One, the method of special studies by an investigator who collects all the seismograms for some one or few well-recorded earthquakes and makes his own readings, posesses certain very considerable advantages. First, the results are less dependent on the variability of seismic velocities from region to region than the method of averaging data from many earthquakes; this advantage is reduced by the fact that the variation is small. Second, the investigator may be considerably more skillful in reading seismograms than those in charge of some of the stations which supply routine seismogram readings; he is likely also to devote more attention to the reading of the less obvious phases. He may be in many cases more, in others less, influenced by his expectations.

The method of averaging readings from many earthquakes possesses evident advantages due to the greater number of data available to be averaged; but it presents new statistical problems, for a reading of an earthquake-phase is subject to errors from various sources, each following a law of its own, the sum not being normal. Not the least troublesome source of error is the arbitrary choice available to the reader in deciding what phases to measure. This difficulty is particularly acute in the case of earthquake-phases which appear to grow gradually out of nothing (as most, fortunately, do not.) These sources of error have been studied by Jeffreys and Bullen, and methods for taking account of them have been worked out [82, 83].

The problem is complicated by the existence of supplementary information on the practices of the various stations. A station which obtains a time-correction once a day is in a different class from one which obtains one only weekly. Some interesting facts have been brought out by the statistical studies of Jeffreys, such as the superiority of the short-period Benioff seismographs, though this is not a very great triumph for the statistical method, inasmuch as this superiority is apparent by glancing at the seismograms. The combining of auxiliary information, where available, with the values of the residuals to obtain the reliabilities of the stations presents delicate problems in weighting.

Jeffreys, Comrie, and Bullen [82] have discussed also the problem of smoothing the averages adopted for various distances in order to obtain corrections leading to new travel-time tables. The principal difficulty here arises from the possibility that the curve may have discontinuities in slope, as indicated in Section 1.3.

The use of the best methods is especially worth while in studies of near-by earthquakes on account of the scarcity of the data and the fact that the interpretation of the distant observations depends on the near-by. Methods of estimating focal depth from macroseismic data, though necessarily crude, may be of value in bridging the gap between the few regions (southern California, Europe, and perhaps New England and Japan) where data are good enough to permit the instrumental determination of accurate depths of focus, and other regions where little more than the non-instrumental effects may be available.

4.2 Periodicity--The periodicity-problem is one of the most intriguing branches of seismology. The studies of Davison [84] and others have brought out many apparent periodicities, some of which are doubtless real, and many interesting relations between them. But the data have not yet been studied in a way to separate the effect of the smoothness of the earthquake-frequencies (due to after-shocks) from the effects of periodicity. Methods for dealing with such problems have been given by Bartels [85], Jeffreys [86], and others. Failure to take account of this effect is the chief weakness of earthquake- and many other periodicity-studies thus far made, but this fault has been mitigated by comparisons of phase between different series of data. Another source of trouble is the fact that microseisms are stronger, and earthquakes therefore more likely to escape instrumental location, in winter than in summer. Many other seasonal and diurnal effects exist.

Some aspects of the problem have received very little attention thus far; for example, the problem of quasi-persistence, which is very important in some geophysical phenomena [85]. Studies of earthquake-periodicity to be most effective should be accompanied by investigation of the forces which may precipitate or retard the occurrence of earthquakes--water- and snow-loads, barometric gradients, forces of cosmic origin, etc. [7, 87, 84, 88 (p. 168)].

The study of earthquake-periodicity suffers much from the inadequacy with which the data have been assembled. Most of the instrumentally located earthquakes are available in the International Seismological Summary, but the instrumental data cover a period of only about 30 or 40 years, which is too short a time for the study of many of the interesting suggested seismic periods. The largest catalogues of earthquakes are so extensive that they have never been printed, and it is doubtful that they ever will. Even after a list is printed, there remains a considerable amount of work to be done in extracting from it the data that may be pertinent in a periodicity-investigation. The only satisfactory solution will be to assemble the data on punched cards. This should be a somewhat easier task than the preparation of a printed catalogue, and will make the data for the first time really available for statistical study.

To return to the more statistical theoretic considerations bearing on the periodicity-problem, there is another aspect of the matter that may have considerable importance. To investigate the reality of a suspected periodicity one customarily computes harmonic coefficients for the period in question and compares them with their distribution, the latter calculated as by Schuster or (less frequently used though usually needed) the modifications already mentioned to take account of smoothness, etc. What (beyond the weight of custom) are the reasons for the choice of harmonic coefficients to be investigated?

Fourier analysis has always attracted mathematicians on account of the elegance of the theory to which it leads. There is little of statistical value in such properties as completeness and orthogonality. A more cogent reason for the use of harmonic analysis is the fact that many natural phenomena are approximately representable as sums of a few sinusoidal terms. Of less theoretical import, but inescapable for practical purposes, is the fact that Schuster's distribution of the Fourier coefficients is almost the only thing available. There is little reason to suppose that earthquake-frequency should vary sinusoidally. In fact, if the earthquake-generating force be proportional to the excess (when positive) of a sinusoid over some constant threshold-value, the variation might be very far from harmonic. The same is true of the type of variation of the frequency of aftershocks suggested by Jeans's [70] study of surface-waves.

These reasons for the use of harmonic coefficients in testing the significance of a periodicity will be seen not to possess any very high degree of universality. Periodicity theory badly needs to be emancipated from this dominion of Fourier analysis. We must use harmonic analysis because some phenomena vary that way, and because no distributions for detecting periodicity but Schuster's have been worked out; let us work out some other distributions.

Theoretically we can derive the distribution of practically any statistic we please to define [89], but the work may be very long and somewhat difficult. It is preferable to face the difficulties rather than be persuaded to the use of methods in which the price of the ready availability of a formula is its great weakness by comparison with the best that could be obtained. Much of the difficulty in deriving distribution-functions arises from complexities such as can be alleviated by the mechanical methods to be discussed in Section 5. It is also noteworthy that the exploratory or even the experimental determination of the distribution of a statistic should never be a long task by comparison with the gathering of the data from which the statistic is computed. For the chief task in this determination is the computation of many values of the statistic; the work of computing each of these, if performed mechanically, should be a very small fraction of the work of gathering the information, which involves the non-mechanical handling of the same number of data as those which enter in the computing of the statistic. The work might be further reduced by a partial theoretical treatment. A distribution once obtained is available for many studies.

Despite these considerations the derivation of distributions is a formidable task, and it is desirable not to define our statistics ad libitum, but with careful regard to the obtaining of the maximum information. The term "efficiency" is used in a special sense in present-day statistics [90], in which the possibility that a statistic may be more "efficient" but less useful than another is hidden in the normality of the population. The deficiency is mitigated by the fact that many populations are approximately normal, while many considerable departures from normality effect only small changes in certain statistics.

Efficiency, or more generally efficacy in detecting and measuring the effect sought, is the prime requisite in defining a statistic to be studied. What considerations are available to guide the choice? Inevitably these must depend on past experience. Prior information enters in some statistical theories explicitly in the form of a prior probability function [91, 83] and in others [90, 92] less obtrusively in the design of experiments, the specification of populations, the (sometimes arbitrary) choice of iso-P surfaces, etc. (Experimenters who assume that their apparatus will obey the Newtonian laws of mechanics do not usually consider it worth while to notice that the assumption involves prior probability.)

We proceed to a fuller discussion of the (more attractive if practicable) explicit treatment of prior probabilities.

4.3. The assessment of prior probabilities--Fisher [90] thinks of likelihood as something altogether dissociated from probability, but from the point of view of Bayes, presently to be considered, the likelihood is simply one of two factors, the other being the prior probability, the product of the two being the posterior probability of any proposition in question [83]. The use of probability has suffered from several objections, which must be considered if the method is to be of use in guiding the choice of the statistics which we define.

There are several difficulties of comparatively minor import for the purposes of inductive science: the paradox that zero-probability need not imply impossibility (which can always be obviated by use of a suitable number system for the expression of probabilities); the fact that several somewhat disparate definitions of probability have been used (I do not fully share Jeffreys's concern about Fisher's "hypothetical infinite population") [91,(p. 220), 83 (pp. 11 and 306)]; and the difficulties encountered in defining a random sequence (it may be that the word "random" is not applicable to a sequence, but to a process).

To proceed to more practical matters, an outstanding objection to the use of probability in preference to likelihood is that the assessment of the prior probability, involving a large body of indefinitely comprehended information, may be done incorrectly, or even wishfully, and will often enable various workers to obtain different results from the same experiment. (Statistics is notorious for this, though much of the trouble encountered in the past has been due to other causes than the incorrect estimation of prior probabilities.) The difficulty will be more acute, of course, in a small experiment than in a large one.

Jeffreys [83] remarks that vaguely remembered information can contribute little to the result in an experiment of any reasonable size and that it is better to ignore it. He provides, then, a number of general-purpose formulas to express the prior probability; if these are used, the results obtained are nearly the same, at least for large samples, as those which Fisher obtains by methods which also purport to be free of prior influence.

These methods meet the objection about the disparate deductions from the same experiment, at least for large experiments, but at the cost of depriving the science of tools which may in some problems be much more effective than those retained; an instance is the possibility of non-sinusoidal variation of earthquake-frequency mentioned in the last Section. The price is too great. Statistics must be made as powerful as possible; if this entails the use by one worker of methods which others do not agree to, then each may supplement his own estimates with a computation made by methods more generally accepted as standard. But it is quite possible to have the advantages of guidance by prior considerations in defining statistics without very much risk of being tempted to use a prior probability so obtrusive that it will vitiate the whole calculation.

This matter is especially acute in the case of experiments which cannot be repeated at will. Astronomical events and most geophysical phenomena are beyond our control. Economic depressions and wars are costly experiments. In some of our most important problems it is necessary to seek the best statistical conclusions from a small number of instances, and here the prior probability has an important rôle.

Jeffreys does not specify his prior probabilities arbitrarily. They are based on a number of rules, which are, for the most part, reasonable enough (though his rule 6, that the number of postulates should be reduced to a minimum [83], would seem to be of small worth in inductive--or deductive--logic).

We shall be led to a natural choice of the prior probability by considering another objection to the use of probability, this time a philosophical one. The prior probabilities used in interpreting an observation made today are those available from previous experience; that is, posterior probabilities resulting from earlier observations. What prior probabilities were then available? By repeating the inquiry we are led to ask what prior probabilities are appropriate to the interpretation of the first observation in the experience of an individual or of a race. Does the theory of probability demand the existence of a priori knowledge?

The difficulty seems to be due to the infinite range on which the probability is defined, for the initial probability on a finite range may be taken to be constant, and any error in this assumption will be quickly overcome by the observations. But it appears that this difficulty may not be very serious after all. If the questions under investigation are stated in comprehensible terms--that is, in terms only of observations possible to be made--the range of the initial probability will be no greater than the range of sensation, which is finite: limited both with respect to intensity (Fechner's law failing at the ends of the range) and discrimination. For example, a straight line considered visually is finite in extent, being of the same size as a semicircle.

If, then, one wished to interpret an experiment in a manner wholly unbiased by previous experience, the natural procedure would seem to be to adopt a probability constant on the finite range of possible sensation. The method is perhaps never used in this perfectly primitive form, but it provides a principle which makes possible the interpretation of experiments in which any amount of prior information is available.

The principles of induction, if stated in fundamental terms, are perhaps the same for all branches of knowledge, but in view of the complications which appear, it is not inappropriate to use quite dissimilar methods in different types of problem. Thus there are two very different cases, both of which pass under the name of "significance test". An investigator may be concerned with the need for the introduction of a new parameter in a physical law, or, on the other hand, with the inquiry whether some quantity, already presumed not to be exactly zero (for example, the excess of the depth of an earthquake over normal) is positive or negative. (Mendelian problems and others present an intermediate case, in which Jeffreys's equipartition of the prior probability may not be appropriate.) Jeffreys is concerned principally with the first type of problem; it would be desirable also in the second to work out tables, using suitable prior probabilities, for the posterior probabilities resulting from the various possible sets of observations (expressed in terms of suitable statistics). (It is an interesting commentary on the insignificance of the prior probability when the number of observations is large, that Jeffreys's results for the first type of question should agree [83, p. 145 and 324] with those resulting from Fisher's use of the likelihood obtained on the null-hypothesis in the second.) Prior probabilities agreeing more nearly with belief, even though they be symmetrical, would be more appropriate to this second type of problem than the null-hypothesis. A consideration of the relative benefits to be derived in accepting or rejecting a hypothesis in all possible cases is essential in fixing "significance" limits, though a little simplification is desirable here for practical purposes.

5. Mechanical calculators

It has appeared that all branches of seismology lead to mathematical problems which we know how to solve in principle, but which become very difficult on account of their complexity. This is true of the problems arising in the study of seismic waves, the equation of state in the interior of the Earth, the theory of complex types of seismographs, the evaluation of the destructive effects of earthquakes, and the determination of the distributions of new statistics.

To say that we can solve a problem in principle means that we can construct a machine to solve it, but there are not many individual problems of sufficient interest to justify (for the present) the construction of very complex machines. The situation is saved by the fact that certain types of machine already in existence can be adapted with comperatively little development work to meet many of the needs of seismology. These machines may be classified broadly into two types, which we may call the mechanical models and the counting devices.

5.1. Mechanical models--Under this term we include all devices designed to represent in a mechanical or electrical way the variation of the magnitudes under investigation. Scale-models and seismographs on shaking-tables [93] or otherwise activated, galvanometers, electrical networks [94], integraphs, and differential analyzers [95] all come under this heading, as well as some machines for solving linear equations.

The possibilities of these machines may be readily appreciated, and will be illustrated in the case of one problem of considerable current interest:

We have seen at the end of Section 3 that an important analysis of an earthquake for the purpose of studying its destructive effects is provided by the family of response-curves obtained by varying the period and damping of a simple oscillator subjected to the motion. One such response-curve can be computed by hand with a reasonable amount of effort, but the computation of a two-parameter family of such curves for each component of every seismogram obtained in an extensive study becomes unwieldy. The differential analyzer is more than adequate for this problem, but the seismograph equation, $(ax'' + bx' + cx) = y''$, occurs so universally in physics and engineering (for example, it provides a generalization of the Fourier analysis) that it is desirable to have a special machine for this equation, such as, for example, the galvanometer being used by Ruge at the Massachusetts Institute of Technology. The equation relating the motions of two seismographs subjected to the same ground motion, namely $(a_1x_1'' + b_1x_1' + c_1x_1) = (a_2x_2'' + b_2x_2' + c_2x_2)$, is also important and can be treated by a slight extension of the same methods.

The scansion of the earthquake-motion is an important part of the problem. Two methods are available, according as the input is, or is not, controlled in part by the response of the mechanism. Photo-electric tape seems likely to prove very useful, especially in the case of electrical networks used to represent complex oscillators subjected to the motion.

5.2. Counting devices--The prototype of the counting device is the mathematician equipped with pencil and paper; we include here all machines which perform the same operations.

Machines for performing the processes of addition, subtraction, multiplication, and division are commonly not provided with means for recording the results of the operations and transferring them to the input of the next machine. Such means are provided in the machines which operate by use of punched cards.

There are many important seismological problems capable of being solved by these machines, including some of those mentioned in Section 5.1. The providing of a resolvent kernel on punched cards for the integral equation involved in computing the depth-velocity relations from seismological travel-time tables should be well worth while. Periodicity-studies (and, indeed, almost all statistical work) are greatly aided by the use of cards.

A set of cards represents a class rather than a function, though it also serves the latter purpose if the cards are kept in order. Where a function is to be represented, it is often desirable to use a tape. The input for Stumpff's harmonic analyzer [96] is punched on tape. In case of complex problems rapid scansion becomes very desirable, and film on which the data are represented by blackened squares or hexagons, and read photo-electrically or photographically, can be employed to excellent effect.

The use of punched cards offers a means of cataloguing by which mathematical and seismological (and other) literature will become enormously more available than it has ever been before.

The foregoing are the more obvious applications of mechanical methods to the problems of mathematical seismology. But there are many other mathematical calculations which are understood in principle, and which could therefore be relegated to machines. Algebraic manipulation is an instance; it should be easy enough to perform such processes by use of cards. But may not also the subtler problems of the calculus be treated, at least in part, in the same way?

The distinction between algebra and analysis has become increasingly hard to perceive in recent years. Boolean algebra (including infinite sums and products--that is, the Prädikatenkalkul) is capable of serving as a basis for the statement of wide classes of mathematical problems. Systematic methods are known in this subject; I have given one [97], and it is not unlikely that every completeness proof for the Prädikatenkalkul could be made to yield a systematic method of investigation. When we consider the length of time that the concept of uniform convergence had been in existence before, for example, the Moore-Osgood theorem was noticed, it appears a not altogether trivial fact that theorems such as this can be readily derived by routine methods available in the Prädikatenkalkul.

With regard to the mechanization of these processes, this can easily be done in finite Boolean algebra. Each term in a Boolean polynomial is represented by a film on which a pair of squares or hexagons is assigned to each letter in the expression, and the films are combined photographically to obtain new terms. The mechanization of constructions in the Prädikatenkalkul seems a little more unwieldy but is quite possible. These mechanical methods will in time free mathematicians from the burden of many long and uninteresting computations, while enhancing the more truly mathematical interest of the problems treated.

It seems possible enough that the methods of symbolic logic may provide a means by which mathematical theorems can be classified, to the great benefit of all branches of applied mathematics.

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A SEISMOLOGIST'S DIFFICULTIES WITH SOME MATHEMATICAL THEORY OR THE LACK OF IT

Perry Byerly

Influence of boundary on speed of wave

The elastic-wave theory is set up for an infinite isotropic elastic medium. Some work has been done on boundary-waves at the interface of two semi-infinite isotropic elastic media. Now most of such theory silently postulates that the elastic properties of a medium retain right up to its mathematical boundaries the same values which they have in its remote interior. We have all the assumptions of geometric optics that wave lengths are short with reference to the length of path in the medium, and that "discontinuities" are "sharp" with reference to wave length. An approach to the problem has been made in the paper "The effect of sharpness of discontinuities on the transmission and reflection of elastic waves," by K. Sezawa and K. Kanai [see 1 of "References" at end of paper].

In seismological work the wave-length is often of the same magnitude as the thickness of a layer which the wave traverses; sometimes the most pertinent transfer of energy is very near the boundary of a medium, that is, within a wave-length of it. In such cases a theory which makes no mention of wave-length is inadequate.

Dr. Harvey Hayes of the United States Naval Research Laboratory made an experiment a few years ago which I consider quite edifying. Referring to Figure 1, between two like steel disks he cemented a mosaic of quartz crystals. By application of an alternating electric current the crystals were caused to expand and contract synchronously in consequence of their piezo-electric properties. By varying the applied frequency the system could be thrown into resonance, and when in resonance sand-particles sprinkled atop the disk were disturbed. When vibrating in its fundamental the oscillating condition is shown schematically by dotted lines, a. It was found that the sand-particles about the outer edge of the disk were excited at a slightly lower frequency than those at the center. This indicated a lower speed of sound in the peripheral portions of the disk than in its interior. For the fundamental, $H = (\lambda/2)$, where λ is the wave-length.

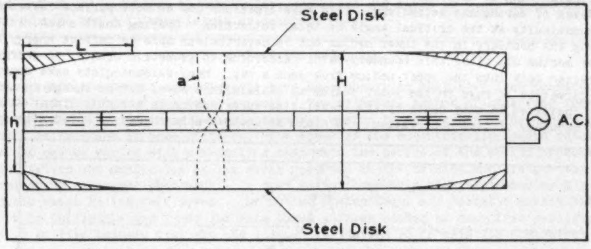

Fig. 1--Oscillation of sand-particles in a system vibrating in resonance

Hence $H = (vT/2)$, where v is the speed and T is the period of motion. A lower frequency, that is, a larger period, requires a lower speed. Dr. Hayes proceeded to cut down the corners of the plate, as shown by the shaded portions in Figure 1, until all the sand on the surface was

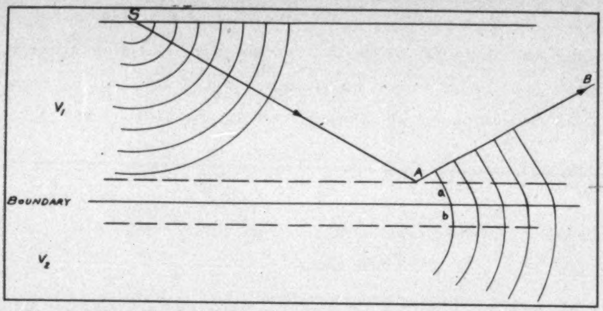

Fig. 2--Refraction of wave-fronts at the boundary of a refractory medium

excited at the same frequency. He found experimentally that the value of L, as shown in the Figure, had to be about one wave-length of the sound used. (The value of h, the thinned-end thickness, was computed from the relation

$$(h/H) = \sqrt{E/(K + 4\mu/3)}$$

that is, the ratio of speed of sound in a rod to that in a solid, where E is Young's modulus for steel and K is the bulk-modulus and μ is the coefficient of rigidity.) For this case the speed of propagation of longitudinal waves was affected by the presence of the boundary to a distance of one wave-length from that boundary.

No doubt the speed of any wave is influenced by the presence of a near-by boundary. And it is very probable that the distance from a discontinuity to which the speed of a wave is affected is a function of the wave-length. At first sight this seems odd, for why should the distance from the boundary to which elastic constants are affected by that boundary be dependent on the tool with which we investigate it? Perhaps a competent theory of elasticity would show in the expressions for the speed of waves an interdependence between the elastic constants, the distance from the boundary, and the wave-length.

The particular kind of wave which I have in mind is the "refracted wave" of prospecting and the similar waves of earthquake seismology. This wave traverses one or more surface-layers and strikes a discontinuity at the critical angle of total reflection. Obeying Snell's law, it is refracted along the boundary in the lower medium but is nevertheless able to refract energy back into the upper medium all along this boundary-path. According to geometric optics, no energy could be refracted back into the upper medium from such a ray. Many seismologists have demanded a very small curvature of rays in the lower medium to explain this wave, but no change in speed with depth is indicated in many cases by the travel-time curve, which is strictly linear within the limits of accuracy of time-observation. To claim an unobservable change of speed with depth in order to preserve geometric optics seems questionable. But if we allow the speeds of waves in the lower part of the upper layer and the upper part of the refracting medium to be influenced by the presence of the adjacent medium, we get a rational explanation of the refracted wave. Figure 2 demonstrates the wave-fronts for such a case. These waves would show an apparent

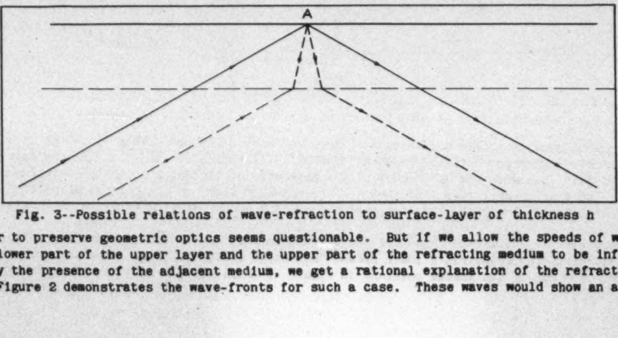

Fig. 3--Possible relations of wave-refraction to surface-layer of thickness h surface-speed equal to the speed in the lower medium. The abnormal curvature of the wave-fronts would probably exist in each medium to a distance about one wave-length from the boundary.

Muskat's paper, "Theory of refraction shooting," [2] is a very important contribution to seismology and again points out to us that our older solutions of the differential equations were after all only particular solutions. The paper explains the refracted wave in a sense. I once heard Dr. Horace Lamb say in a lecture at Cambridge, "We put our physical observations into an equation and then we go into higher mathematics with it and come out with another equation which leads us to conclusions, and we do not know how we got them." Perhaps we do not need to know, but I do not feel satisfied until I have some physical picture.

The question as to what thickness of a transmitting layer is necessary before we can apply Snell's Law is important. What must be the thickness in relation to wave-length before we can compute the refraction-angle i_2 in the layer from $\sin i_2 = (v_2/v_1) \sin i_1$, i_1 being the angle of incidence on the layer? In particular, we may wish to compute the energies reflected at the Earth's surface as longitudinal and transverse waves when a longitudinal wave is incident. This is a function of the angle of incidence. If the Earth's surface is covered with a layer of thickness h, composed of rock of speed lower than that in the mantle below it, for what wave-lengths must we compute the path of the wave as broken by refraction according to Snell's Law at the bottom of the layer (see dashed path in Fig. 3), and for what wave-lengths may we neglect this break in path (see solid path in Fig. 3)? Certainly if the wave-length is a small fraction of h, we must consider the break. But if the wave-length is say 10h there is probably no reason to consider the layer as affecting the angles of incidence and reflection at the surface. But what about intermediate values of wave-length?

Of course, if we treat the beginning of a wave-group, we might think of it as an "impulse" and therefore an infinitely short wave, and then as far as computing travel-times goes, we could use Snell's Law. But our first wave is often 70 km long and if we are measuring its maximum amplitude, we have surely departed far from an "impulse."

A similar problem arose in Benioff's theory of his strain seismograph [3]. This instrument consists of two piers 20 meters apart. To one is clamped a long rod or pipe which extends to the other pier but is not fastened to it. On this end of the pipe is attached part of an electrical transducer, while to the pier are fastened the other parts of the transducer. Relative motion of the two piers is desired. The question is, by how much time does the motion of the farther end of the pipe lag behind that of the end clamped to the first pier? For an "impulse" we agree that the farther end of the pipe will lag in response by the time taken by a sound-wave to traverse the pipe. But for sustained motion the problem alters. Benioff calculates that if the clamped end of the rod moves as sin pt, the unclamped end moves as a $\sin pt \sec 2\pi L/\lambda$, where L is length of pipe and λ is wave-length of recorded wave.

Free periods

For some years the German seismologists have been observing what have been considered to be "free periods" of portions of the Earth's crust. The principal method of observing them is to set up a ground-shaker which imparts simple harmonic motion to the ground. The ground-motion is observed on a seismograph at some distance. The speed of the eccentrically loaded rotor which causes shaking may be varied with a resulting change in the period of the motion produced. For certain periods the amplitudes of the waves recorded on the seismograph are greater than for others, indicating resonance-phenomena. In some cases these have been explained as a reinforcement of direct waves by indirect waves. The United States Coast and Geodetic Survey has done similar work in Califronia and finds for many cases a large number of such free periods; for example, four or five between 0.01 sec and 1 sec. They stand in no relation suggestive of harmonics.

It is difficult for me to see how so many free periods can exist. Even if it seems reasonable for a surface-layer to have a free vibration in particular cases, such as a limestone layer underlain by clays, it seems odd that all localities, alluvial flat-lying land included, should show so many. If successive layers have free periods, the whole should be a coupled system with no resultant free period. Perhaps the successively lower layers are sufficiently larger than those above them completely to dominate once they are set in motion.

Surely resonance in a layer can be no more than a reinforcement of the source by a wave reflected at the bottom of the layer. Then if there is no change in phase at the bottom of the layer of thickness h, the fundamental free period is $T = (2h/v)$, where h is the layer-thickness and v is the speed of propagation in it; whereas, if there is change in phase, $(T/2) = (2h/v)$. This postulates vertical paths for waves. Earthquakes from a distance could not give rise to this directly but might cause a similar effect where the resonance-period would depend on the angle of incidence of the wave.

In the two cases mentioned, $\lambda = 2h$ and $\lambda = 4h$, respectively, and there seems to be a question as to whether the v just used should be the speed of waves in a medium of the material of the layer but of large extent; that is, for longitudinal waves the speed of sound in the layer-medium. In some of the German work [4] in which a shaker is so used and the layer set into simple harmonic motion, it is assumed that the difference in the phase travel-time of the direct and the indirect (or refracted) wave is $x[(1/v_2) - (1/v_1)]$, where x is the surface distance between shaker and seismograph, and v_1 and v_2 are speeds of propagation in the layer and underlying medium, respectively. This would mean that, once set in vibration, the top and bottom of the layer moved synchronously. There seems to be some question as to this matter, and we see it is related to the problem discussed above. (The travel-time of the direct wave through the upper layer is (x/v_1), while that of the refracted or indirect wave is $[(x/v_2) + 2h(\cos i/v_1)]$, where $\sin i = (v_1/v_2)$.

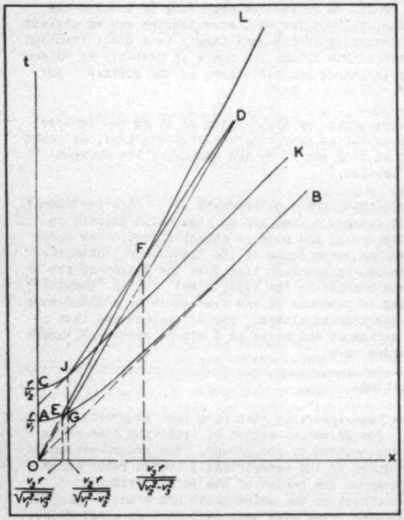

Fig. 4--Nakano's theoretical travel-time curves

S-waves

Elastic-wave theory tells us that, in an infinite isotropic elastic solid, a heterogeneous source will generate two types of elastic waves which propagate with different speeds, namely, a purely longitudinal wave of speed $v_p = \sqrt{(K + 4\mu/3)/\rho}$ and a purely transverse wave of speed $v_s = \sqrt{\mu/\rho}$, where K is the bulk-modulus, μ is the coefficient of rigidity, and ρ is the density. An elastic theory for the Earth for which the elastic constants vary with depth would be useful, but it is not a simple problem. Love [5] made a short step toward it when he gave his theory for a compressible planet, but his theory did show that even a slight departure from the isotropic case rendered the two types of wave no longer completely separable. Birch [6] has recently applied Murnaghan's elastic theory to the problem of the effect of pressure on seismic-wave speeds.

If the source were not heterogeneous but a "perfect" explosion in an isotropic medium, we might expect only longitudinal or P-waves to be generated. However, in any actual case there must be some transverse or S-waves set up. Is the general lack of observation of S by seismic prospectors due to the weakness of shear-components in the source, or perhaps due to frequency of S-waves not lying in the range to which their receivers are sensitive? On seismograms of earthquakes S very frequently has a longer period or lower frequency than P. Wiechert noticed that the ratio of the periods of S and P (in the same record) are frequently inversely as the ratio of their speeds of propagation. He suggested that this might be explained if the wave-length, rather than the period, were controlled by the source. This would seem to require that a portion of a fault, say, equal to a wave-length, break simultaneously at all parts. Such a break seems most improbable. A reflected S due to incident P is weak if the angle of incidence is small, so transformed waves should not be very conspicuous in reflection-prospecting. Is it likely that surface-formations have too low a coefficient of rigidity to transmit S well? This might show up in prospecting, where the wave-lengths are very short, but not in earthquake-observations with longer waves.

Nakano waves

Nakano [7] in a very elaborate discussion of elastic-wave theory for a bounded medium concluded the existence of two kinds of boundary-waves not previously so deduced. Figure 4 shows his theoretical travel-time curves. AB is the travel-time curve of the direct longitudinal wave of P, and CD is that of the direct transverse wave or S. EFL corresponds to the Rayleigh wave, and GD and JK, respectively, show transverse and longitudinal waves which are propagated along the surface. Note that the incidence of transverse waves at the surface from the focus sets up a longitudinal wave which travels along the surface. And the incidence of longitudinal waves at the surface sets up transverse waves which travel along the surface; these would have to vibrate in the vertical plane if set up by incident longitudinal waves. Such waves do not emerge from the older theory. More specifically, the possibility of the existence of the first type is denied by simpler theory. I should like to see some mathematician check this work of Nakano.

Elastic rebound

Sezawa [8] tells us: If a force is applied very slowly to a portion of an elastic body, the total energy supplied will be stored in strain-energy. If it is applied very rapidly, only one-half the energy applied is stored as strain-energy, while the other half will be dissipated as elastic waves. Now in the case of elastic release of strain-energy we have the same dynamical problem as if the force which caused the strain-accumulation still existed but we applied an equal and opposite force. The problem is then the one we had before, he says. If the opposite force causes slow return to elastic relief, we put in work just equal to the strain-energy. But if the opposite force causes very quick relief, we must put in twice the energy of elastic strain, and half goes out as elastic waves. (I trust I interpret Sezawa correctly). Therefore, Sezawa says, "If the pressure is released very quickly, the energy applied to the origin is equal to twice the value of the statical strain-energy. ... It is thus impossible to evolve wave-energy from the strained part of an elastic body, even in sudden release of the force at the origin. Half the value of the energy at the origin, however, is consumed in resolving the statical strain, the remaining half alone being transmitted as elastic waves."

This would destroy the elastic-rebound theory of the cause of earthquakes. It seems to me to neglect the fact that, on rupture of a fault, the original force causing the strain which resulted in rupture is removed and replaced by a much smaller force of sliding friction and that the strain-energy is quite competent to overcome this and send out waves too. Waves are probably sent out by local vibrations along the fault while the friction is being overcome, that is, by local strains quickly accumulated and quickly dissipated. Surely laboratory and engineering experience is contrary to Sezawa's conclusion. Sezawa's physical argument is amplified by a mathematical treatment of the question.

Integrating seismograms

Leet [9] in his book says that serious error has been made by workers who have integrated seismograms in that they have integrated continuously, neglecting the fact that on the arrival of a new phase the velocity of the seismograph-pier experiences a discontinuity, a sudden finite change in velocity, and that the seismograph-pendulum records a line which has a discontinuity in slope. Such would call for new initial conditions. To quote Leet, "The variable initial conditions, however, cannot be neglected, and a continuous integration of the curve $y = F(t)$ (the seismogram) without considering the singuler points of the function $x = f(t)$ (earth-displacement) is quite inadmissable." To me it is highly distasteful to think of a discontinuity in the speed of the pier with the necessary infinite acceleration. I should say that, if you speed up your recording drum, you will find that any sudden change in slope on the seismogram becomes a gradual one. Indeed, on modern seismograms I recollect seeing no sharp corners save perhaps at crests of waves where drum-rate is slow.

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AMERICAN GEOPHYSICAL UNION

REGIONAL MEETING

AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

December 1939

PART IV-B

SYMPOSIUM ON HYDROLOGIC PROBLEMS IN THE OHIO AND MICHIGAN BASINS

Columbus, Ohio
December 29, 1939

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SYMPOSIUM ON
HYDROLOGIC PROBLEMS IN THE OHIO AND MICHIGAN BASINS

[A symposium presented under the joint auspices of the Section of Hydrology of the American Geophysical Union, the Section of Geology of the Ohio Academy of Science, the Geological Society of America, and Section E of the American Association for the Advancement of Science at Columbus, Ohio, December 29, 1939]

PREFACE

Howard A. Meyerhoff

The significance of water-supply in human economy is tacitly assumed, but the broad range of specific hydrologic problems upon which the economy of any region depends is but vaguely appreciated, even by those who are actively concerned in their solution. When plans were being made for the Columbus meeting of Section E in December, 1939, it was R. A. Smith of the Michigan Department of Conservation who stressed the importance of hydrology in Michigan and in the Ohio Drainage-Basin. The emphasis which he placed upon the subject led to a symposium sponsored jointly by the Section of Hydrology of the American Geophysical Union, the Section of Geology in the Ohio Academy of Science, the Geological Society of America, and Section E of the American Association for the Advancement of Science.

The six articles which follow are condensed versions of papers which were read at the meeting held on the afternoon of December 29, 1939. In the preparation of these articles for publication, each of the authors has sacrificed much of the interesting detail which his original paper contained, but there remains a body of material that amply demonstrates the scope of hydrologic research which is being applied to urgent problems in the eastern and northern interior sections of the country.

C. L. McGuinness sketches ground-water conditions in Michigan, in part as they have been revealed by a search for local water-supplies which can be utilized in fighting forest-fires. But his sketch leaves no doubt about the vast significance of ground-water in a State where three-fourths of the population are dependent on subsurface supplies. Fred H. Klaer, Jr., discusses the vital question of ground-water depletion in southwestern Ohio, where many industrial enterprises are dependent on the maintenance of water-supply, and where further industrial expansion hinges upon the additional subsurface water-reserves which may be available.

Occupying a key position in all types of hydrologic research, whether it be on ground-water, runoff, floods, or soil-erosion, is the Weather Bureau; and the rôle which this agency is assuming in hydrologic work is well presented by Thomas S. Southwick. All too briefly Major Lewis A. Pick outlines the varied problems which must be solved if the Ohio Valley is to be provided with a measure of protection against devastating floods; and C. C. Chambers, in discussing a project in which flood-protection and water-conservation have already been achieved, modestly points out how much must still be learned if all the possibilities of the Muskingum Project are to be fully realized. Finally, William F. Simpson and J. S. Cutler effectively demonstrate that every hydrologic problem is minimized, and some are partly solved, if precipitation falls upon lands that are properly and intelligently used.

Without question the greatest value to be found in a symposium on the "Hydrologic problems in the Ohio and Michigan basins" is not so much the individual and collective merit of the six papers composing it, as it is the inspiration these papers will give to members of the several professions concerned. Surely the latter will be inspired to see and to study the whole field of hydrology more broadly, and to apply such principles as have been discovered or evolved in solving problems peculiar to Michigan and to the Ohio Basin, in other regions where comparable hydrologic questions are equally acute.

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A BRIEF REVIEW OF GROUND-WATER CONDITIONS IN MICHIGAN

C. L. McGuinness

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Geology

The State of Michigan makes up about one-half of the area of the great Michigan Synclinal Basin, the remainder of which embraces Lakes Michigan and Huron and small parts of Wisconsin, Illinois, Indiana, Ohio, and Ontario [see 1 (p. 7) of "References" at end of paper]. The Basin has characteristics of both a geosyncline and a major structural basin. The geosynclinal origin is indicated by the facts that the Basin has been progressively downwarped, the beds thicken markedly into the central area, the outline of the course of the outcropping rocks is roughly oval, and the minor structures within the Basin are mostly parallel to the longer diameter of the downwarp. Evidence of several periods of isolation and evaporation and the absence of thick series of coarse clastic sediments in the post-Cambrian rock-column are features that are more characteristic of structural basins.

Underlying the region is a basement-complex of igneous and metamorphic rocks of pre-Cambrian age [1, p. 19]. Above these rocks is a succession of Paleozoic sedimentary formations, ranging in age from Cambrian to Carboniferous, dipping toward the central part of the Basin. The total thickness of sedimentary rocks in the central part of the Basin is probably more than 12,000 feet [1, p. 21]. Within the State of Michigan the crystalline rocks emerge from beneath the Paleozoic sediments only in the western highlands of the Upper Peninsula [2, geol. map]. The eastern part of the Upper Peninsula is a belted lowland underlain by sedimentary rocks ranging in age from Cambrian to Silurian [3, p. 48].

The consolidated rocks of the State are nearly everywhere covered by glacial drift deposited during the Illinoian and Wisconsin stages of the Pleistocene epoch. The drift varies in thickness from a few feet to about 900 feet, and it may be more than 1,000 feet thick in some places in the Lower Peninsula as yet unexplored by the drill [1, p. 63]. The average thickness of the drift in the Lower Peninsula is about 300 feet [3, p. 106]; in the Upper Peninsula the average thickness is considerably less. In the Upper Peninsula the glacial deposits are largely those of the younger or Wisconsin stage of glaciation [4, p. 16]. In the Lower Peninsula the deposits of the older or Illinoian stage make up the greater part of the total thickness of the drift, and the deposits of the Wisconsin stage form a veneer ranging in thickness from a few feet to 100 feet or more [3, p. 116]. However, except in places where the drift is very thin or where strong ridges of the older drift dominate the topography, the Wisconsin deposits determine the immediate configuration of the surface [4, p. 20].

Ground-water conditions

Consolidated rocks--Most of the pre-Cambrian rocks in the western part of the Upper Peninsula are dense and compact, and generally yield very little water to wells. In most places in the western part of the Upper Peninsula the glacial drift overlying the crystalline rocks is sufficiently thick and permeable to furnish at least moderate supplies of ground-water, but in places where the drift is thin or absent the crystalline rocks generally yield small supplies of water adequate for domestic purposes. Only a few communities have public supplies derived from the crystalline rocks [5, pp. 237-239].

In the eastern part of the Upper Peninsula all the Paleozoic formations except the Maquoketa shale, of Ordovician age, generally yield moderate or large supplies of water to wells. The sandstone of upper Cambrian age is the most productive water-bearing rock, but the Ordovician and Silurian limestones and dolomites yield large supplies of water in many places [5, pp. 237-239]. Flowing wells have been obtained in some areas of low elevation from both the Paleozoic rocks and from glacial and lake deposits [3, pp. 97-99].

The most productive bedrock-formations in the Lower Peninsula are the sandstones of the Marshall formation of Carboniferous (Mississippian) age, especially the upper part, which has been designated the Napoleon sandstone, commonly known in Michigan as the Upper Marshall sandstone. The Marshall formation underlies almost the entire central part of the Peninsula, and crops out below the drift in a belt that is roughly a distorted square, the corners of which lie in Huron, Hillsdale, Oceana, and Otsego counties [1, p. 57]. The formation yields fresh water in most places in or near its outcrop-areas, but under thick cover it yields a concentrated brine [1, p. 56] that has been exploited as a source of bromine and calcium and magnesium salts, especially in Midland, Isabella, and Gratiot counties [1, p. 193].

In most places in the Lower Peninsula rocks older than the Marshall formation yield only small quantities of water to wells, or yield water that is too highly mineralized to be suitable for domestic use. Several of these formations yield brines that have been commercially exploited in some places [1, pp. 189-196]. Of the rocks lying above the Marshall formation, the Parma sandstone and the sandstones of the Saginaw formation yield in many places large quantities of moderately good water.

Unconsolidated deposits--Throughout most of the State the unconsolidated deposits of glacial origin form the principal source of ground-water. In places where the drift is thin or absent, or is made up largely of clay, surface-streams or the underlying bedrock must be used as sources of fresh water.

The surficial unconsolidated deposits of Michigan fall into four broad classes: Terminal moraines; ground-moraines or till-plains; outwash-deposits; and lake-beds. The terminal moraines were deposited during the last retreat of the glaciers, at times when the glaciers paused in their retreat or even readvanced briefly [3, p. 55]. A morainal belt swings across the southern part of the western half of the Upper Peninsula [6, geol. map], and extensive moraines were deposited at the edges of the Michigan, Saginaw, and Huron-Erie lobes of the Wisconsin glacier in the Lower Peninsula [3, pp. 115-116]. Areas where two or more lobes of the glacier coalesced are naturally areas of heaviest deposition of moraines. Such areas are found in the Thumb Upland and Northern Upland of the Lower Peninsula [3, p. 114].

The moraines are made up largely of coarse gravel and boulders imbedded in a clayey matrix, and they contain many beds of water-bearing sand and gravel, some of which yield large supplies of water to wells.

The ground-moraines or till-plains were deposited during stages of rapid retreat of the glaciers, when the materials carried by the ice were deposited in a jumbled mass, with little sorting [3, p. 124]. Numerous beds and lenses of sand or gravel occur in the till, however, showing local deposition by running water, and these beds supply most of the drift-wells in the till-areas. The clayey ground-mass is rather impermeable and generally yields little or no water to wells.

The outwash deposits were formed at the outer edge of the ice-border during the development of the adjacent moraines. They make up both extensive plains and channel deposits along the lines of glacial drainage [3, p. 116]. The deposits consist largely of sand and gravel and are capable in most places of yielding large supplies of water to wells. Scattered outwash deposits are found in the Upper Peninsula and extensive deposits are found in the interlobate areas outside and between the terminal moraines in the north-central, west-central, and south-western parts of the Lower Peninsula [3, geol. maps].

Lake-beds deposited in the ancestral Great Lakes surround Saginaw Bay and fringe the eastern edge of the Lower Peninsula from Saginaw Bay southward to the Ohio line. Lake-deposits also occupy large areas in the eastern and northwestern parts of the Upper Peninsula [3, 6, geol. maps]. The beds are sandy in some places and clayey in others. Sandy beach-ridges and outwash deposits occupy small areas in the lake-bed regions. Ground-water supplies are confined chiefly to the sandy surficial deposits and to occasional beds and lenses of sand and gravel found in glacial drift below the lake-clays. The clays themselves are relatively impermeable.

Investigations in the forested areas

The Geological Survey Division of the Michigan Department of Conservation and the Geological Survey of the United States Department of the Interior have made cooperative investigations of the geology and water-resources of the State since the latter part of the nineteenth century. Since the fall of 1932 a cooperative program of ground-water investigations and measurements of ground-water levels has been in progress in the State [7, p. 71]. The program was begun in Roscommon County, where an investigation was made to determine the areas in which ground-water occurs at depths of 20 feet or less, in sufficient quantity to be pumped from wells by suction for use in suppressing forest-fires. The investigation involved the sinking of many shallow wells to determine the depth to the water-table and the occurrence and character of the water-bearing sands in all parts of the area studied. The areas mapped were those in which the conditions are favorable for sinking wells that will supply a fire-hose and in which these wells can be constructed very quickly after the fire has broken out.

In the spring of 1933 the Michigan Emergency Conservation Work was begun, and a number of Civilian Conservation Corps camps were established in the forested areas of Michigan. This made possible an expansion of the investigations to determine areas suitable for the construction of wells for fire-control purposes. Surveying parties composed of members of the Civilian Conservation Corps, headed by geologists of the State Geological Survey or of the Civilian Conservation Corps itself, have since put down about 5,000 test-wells and have completed the mapping of about 60 townships in the forested areas. The maps show the depth to the water-table in all parts of these townships; they also show the areas that are favorable as to water-bearing sand, and the location and extent of areas underlain largely by clay, where the rapid construction of wells for immediate use in fighting fire is not feasible.

Some of the test-wells have been saved for observation-purposes and the depth to the water-level in each of these wells is measured once or twice a month to determine seasonal and long-time trends in the fluctuations of the water-table. Periodic measurements are also being made in wells on the shores of several inland lakes in the northern part of the Lower Peninsula in order to study the interesting and significant relations of the fluctuations of lake-levels to fluctuations of the ground-water table. An automatic water-stage recorder was installed on a well at the Forest Fire Experiment Station in Roscommon in 1934, and has been in operation since that time. Records of water-levels obtained since 1934 have been published in Water-Supply Papers 777, 817, 840, and 845 of the United States Geological Survey [7, 8, 9, 10].

The continuous records afforded by the automatic recorder show many significant fluctuations of the water-table in addition to the seasonal and long-time trends. Among these are fluctuations caused by changes in temperature which result in freezing or thawing of the ground. Little recharge of the water-bearing sands takes place when the ground is frozen, and the water-table generally declines slowly as a result of ground-water discharge into streams or springs. Minor fluctuations caused by changes in atmospheric pressure are sometimes shown when the ground is frozen and the interstitial ice acts as a more or less effective confining bed. A change in temperature that permits thawing of the ground is indicated by a relatively large and rapid rise of the water-table as recharge from melting ice and snow takes place. During the growing season a small daily lowering of the water-table, resulting from transpiration by plants, is generally shown.

The Michigan Forest Fire Experiment Station, under the direction of G. I. Stewart, has given special attention throughout the investigation in the forested areas to the development of equipment and technique for the rapid construction of shallow tubular wells. The equipment and methods used have been described in some detail in a paper by Stewart [11].

The standard method of installing wells consists of three main steps: First, a well casing, usually three to four inches in diameter, is washed in to a depth at least several feet below the water-table, using water pumped under pressure from a tank-truck; second, a well-screen attached to a pipe is dropped inside the casing; and third, the casing is withdrawn, leaving the well-pipe protruding from the ground, ready for attachment to the suction-hose of the fire-pump. The work can be done by two or three men, and no derrick is necessary. The complete process, from the start of the work until everything is ready to throw a jet of water, generally requires only about ten minutes for a well in sand, if no unusual difficulties are met. A few minutes more is required for a well penetrating clay or hardpan.

An alternate method dispenses with the well-casing, and involves the use of drive-points instead of standard well-screens. A hole is washed down to the water-table with the washing nozzle. A well-pipe, to which is attached a drive-point, is then dropped into the hole and driven down until the point is sufficiently submerged. Driving the point into the saturated material below the water-table is generally not difficult, and wells of this type can be completed in sand in about 15 minutes.

The standard method is preferred to the alternate method because well screens of any given size have larger intake areas than drive-points of similar dimensions, and consequently give larger yields. Yields of 65 gallons a minute have been obtained from wells in coarse sand equipped with wire-wound well-screens three inches in diameter and three feet long; yields of about 40 gallons a minute have been obtained from wells in similar material equipped with drive-points of comparable size.

The success of these methods depends largely upon the geologic conditions. The water-table must be less than 20 feet below the surface to permit the use of suction-pumps, which are the only practical type for purposes of this kind. Moreover, the water-bearing material must not be too tightly packed or cemented, and it must be permeable enough to yield adequate supplies to wells of small diameter. These conditions are met in many places in the sandy outwash plains that occupy large areas in the northern part of the Lower Peninsula of Michigan. With these methods supplies of water adequate to fight fires in brush and peat-bogs can be quickly developed.

Public water-supplies

According to a report issued by the Michigan Department of Health [12, p. 46], in which the figures for population are based on the census of 1930, there were in Michigan on January 1, 1938, 339 public water-supplies. Two hundred fifty-five supplies, or about three-fourths of the total number, were derived from ground-water sources; three supplies obtained water from both surface- and ground-water sources. Detroit and some of the other large cities have surface-water supplies, but the cities and towns that have waterworks supplied with ground-water have an aggregate population of about 830,000, and combination surface- and ground-water supplies serve a population of about 35,000.

Of the 255 public water-supplies derived from ground-water sources, about 70 per cent, serving a population of about 530,000, were obtained from sand and gravel of glacial origin; about 25 per cent, serving a population of about 210,000, were obtained from bedrock; about five per cent, serving a population of about 90,000, were obtained partly from glacial deposits and partly from bedrock. Many of the supplies derived from bedrock were obtained from wells in the Marshall or Coal Measures sandstones, or both.

Assuming a per capita daily consumption of 100 gallons, about 83,000,000 gallons of ground-water a day, is produced for public supplies in Michigan. Many industrial concerns throughout the State obtain large supplies from private wells, but few data are at hand to give even an approximate idea of the quantities of water utilized by such concerns. Practically all the million and a quarter people in Michigan not served by public water-supplies use ground-water.

It is obviously important that records should be obtained as to the pumpage and the fluctuation of the water-levels in the cities and towns supplied by ground-water, in order that reliable information may be available as to the depletion of the ground-water supplies and the extent to which they can be drawn upon for additional needs. During the summer of 1939 the cooperative program in Michigan was expanded to include measurements of water-levels and the collection of records of pumpage in a few of the largest cities in the southern part of the State that use ground-water for public supply, including Lansing, Pontiac, Kalamazoo, Battle Creek, and Jackson. Fortunately, the water departments of the first three cities named have made measurements of water-levels in their wells for some years, and these records of former years will doubtless be valuable in future studies of the safe yield of the water-bearing formations.

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GROUND-WATER PROBLEMS IN OHIO, WITH SPECIAL REFERENCE TO THE INDUSTRIAL AREA OF CINCINNATI IN BUTLER AND HAMILTON COUNTIES

Fred H. Klaer, Jr.
(Introduced by David G. Thompson)

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The importance of ground-water as a natural resource to be used for water-supply and industrial purposes is often overlooked by the general public. A recent survey conducted by the Engineering News-Record [see 1 of "References" at end of paper], based on data obtained from State Sanitary engineers, shows that 9,100 out of a total of 12,700 public water-supply systems in the United States obtain water from underground sources. Of the total population served by public water-supplies, about 28 per cent or 22,500,000 people are served by systems using ground-water. In addition to the quantities of water pumped for public supplies, many million gallons of ground-water are pumped daily for rural, domestic, and industrial purposes. In many of the cities that have public supplies from surface-sources, there is nevertheless heavy pumping from private wells for industrial purposes.

In the State of Ohio, there are about 410 public water-supply systems, serving approximately 4,850,000 people [2]. Of these, 319 systems used ground-water to supply about 1,250,000 people. The communities served by ground-water range in size from Dayton, with more than 200,000 inhabitants, to small towns and villages of less than 1,000 inhabitants. Most of the larger cities are served by surface-water supplies. Thus in Ohio the average surface-water system supplies about 35,000 people, while the average ground-water system supplies only about 3,500 people.

Allowing a per capita consumption of 100 gallons of water per day, the daily consumption of ground-water from public water-supply systems in Ohio is estimated to be about 125,000,000 gallons. At ten cents a thousand gallons, the value of this ground-water would be $12,500 a day, or more than $4,500,000 a year. Considering the water pumped from private wells for industrial use, a quantity which is probably equal to, or greater than, that used for public supply, it may be estimated that the total sale-value of ground-water used in the State of Ohio, exclusive of the rural supplies, is about $10,000,000 a year.

Many millions of gallons of ground-water are used daily by various industries in Ohio, such as paper-mills, steel-mills, roofing-factories, and ice-plants. In certain areas, the cost of drilling wells and pumping water is considerably less than that of buying water from a public supply-system, especially where treated water is not required. Most ground-water is too hard to be used for boiler-water without softening, but many industrial processes do not require treated water.

As the temperature of ground-water is generally lower during the summer season and more constant than that of surface-water, ground-water is greatly preferred for cooling and condensing purposes and for air-conditioning. During the past few years, in which many motion-picture houses, department stores, and restaurants have installed air-conditioning equipment, the consumption of ground-water has been greatly increased. Not only has this caused an increased draft on the underground supply, but in some cases has given rise to problems of disposal such as overloaded sewers. In one known case in Columbus, air-conditioning water is being returned to the ground through wells, but this system has not been in operation long enough to determine the effect on temperature or on the return well. The return of such water to the ground has been practiced on a large scale on Long Island and for several years has been under investigation by the Geological Survey.

The most important sources of ground-water in large quantities in Ohio are the preglacial and interglacial stream-valleys, of pre-Illinoian age, which were filled to depths of several hundred feet with sand, gravel, silt, and clay during Pleistocene time. Prior to the glacial epoch, the main drainage in Ohio is believed to have been northward toward the present site of Lake Erie. When the northward-flowing streams were blocked by the advance of the ice, long narrow lakes were formed, which were gradually filled with silt and clay from the streams flowing into them, and with coarse alluvium washed out from the melting ice-front. The lakes found new outlets, and a new drainage-system, similar to that of the present day, was developed. Although often the direction of the streams was reversed, in some places the new drainage coincides with the ancient system, and in other places it is discordant with it. Many of the alluvial-filled valleys were abandoned by the streams which originally drained them. Their deposits of sand and gravel, commonly several hundred feet in thickness, remain as great underground reservoirs to supply water to many municipal and industrial wells.

It is believed by many persons that the ground-water supplies of Ohio have been seriously depleted during the past few decades. Others have reported that no serious decline in water-level has occurred, and there is no agreement as to the amount, rate, or extent of the reported decline. In certain heavily industrialized areas, definite evidence of large declines in ground-water level has been obtained, but the rate of the decline and the extent of the areas affected are not definitely known at the present time.

A Civil Works Administration project was conducted in 1933 and 1934 to gather data on wells and water-levels in Ohio under the leadership of David C. Warner, of the State Water Conservation Board. Data on about 149,000 wells were obtained from parts of most of the counties in the State; but, because of lack of funds, the survey was never completed, and the data collected have not yet been compiled and released to the public. Merle V. Baker, one of the geologists connected with the ground-water survey, made a brief study of the data and concluded that the general ground-water level in most shallow wells throughout the State had declined, but the amount and rate could not be determined. The deep wells of the State were apparently little affected at that time.

Ohio suffered major droughts in 1930, when the precipitation was only 70 per cent of normal, and in 1934, when the precipitation was only 69 per cent of normal. From 1929 to 1934 there was an average accumulated deficiency in the State of 22.6 inches of rainfall. The principal deficiencies in both 1930 and 1934 occurred during the winter and spring when recharge to the ground-water reservoir is generally greatest. From 1934 to 1938, the precipitation was more nearly normal. The Civil Works Administration ground-water survey was made in 1933 and 1934, when the water-levels were doubtless lower than normal because of the deficiency in precipitation.

In order to determine any long-time trend of water-levels, it is necessary to make measurements of water-levels at regular intervals in a considerable number of wells over a period of many years. Water-levels in shallow wells rise and fall with variations in precipitation. The highest and lowest levels do not occur at the same time each year, and hence measurements made only once a year or at irregular intervals are likely to be misleading.

Serious declines of water-level in the deep wells of the State have been reported, but few such reports are verified with actual measurements of water-level. Few records of past water-levels are available, and many of those are not complete. Usually no record is kept of the point from which the water-level is measured, the time of year at which the measurement was made, whether or not near-by wells were being pumped when the measurement was made, and in some cases there is not even a record as to the particular well in which the water-level was measured. Therefore it is difficult to compare past water-level measurements with those made at the present time.

There is urgent need for additional information regarding the rate, amount, and extent of the decline in water-levels throughout the State, especially in areas which are subject to over-development. Accurate measurements of water-level should be made at regular intervals or by means of automatic recorders in properly selected observation-wells, and the records should be preserved and published in orderly fashion, as is now done in many other States. There will thus become available accurate information for determining the natural depletion and replenishment of the ground-water supplies from year to year and for longer periods. If systematic records of pumpage are also obtained, it will become feasible to determine for each heavily pumped area the perennial supply for the area, and to apply effective remedial measures, such as redistribution of the pumped wells and artificial recharge. One of the conditions which in past years has invited industrial concerns to establish plants in Ohio has been a plentiful supply of water that can be recovered at small cost from underground sources. If this supply becomes inadequate or the cost of obtaining water is greatly increased, this inducement for new industries will disappear and some of the existing industrial plants that require large amounts of water for processing may be moved elsewhere, resulting in loss of revenue and employment to the State.

The industrial area of Cincinnati

In the industrial section tributary to Cincinnati, in Butler and Hamilton counties, the water-levels in both deep and shallow wells declined rapidly during the drought of 1934. Local interest was aroused and in 1936 a preliminary investigation of ground-water conditions in the two counties was made by David G. Thompson, of the Geological Survey, United States Department of the Interior, in financial cooperation with the Boards of County Commissioners of Butler and Hamilton counties. The purpose of the investigation was to determine the seriousness of the decline in water-levels and the type of detailed investigation needed to determine the safe yield of the water-bearing beds. The results of the investigation were released in manuscript and were summarized in a mimeographed memorandum in April 1937 [3]. Following the recommendations made in the report, a more detailed study of the ground-water conditions in the area was started in June, 1938, and this investigation is in progress at the present time.

The purpose of the present investigation is to determine as fully as possible the amount and rate of decline of water-levels, the extent of the areas affected, the amount of water that may be pumped perennially from wells without permanent depletion of the supply, and the feasibility of methods for increasing the underground supply by redistribution of the wells or by artificial recharge.

In Butler and Hamilton counties, the principal sources of ground-water are the ancient valleys of the ancestral Ohio River drainage-system during the "deep-stage" period of interglacial drainage. It is believed that, prior to the Illinoian stage of glaciation, the ancestral Ohio River followed its present course westward as far as the mouth of the present Little Miami River, but that at this point the ancestral Ohio swung northward, joining the Little Miami River several miles south of Madisonville, and thence flowed westward through the so-called Norwood Trough, now occupied by Oakley, Norwood, and Bond Hill, Near the present site of St. Bernard, the ancestral Ohio was joined by the ancestral Licking River, which flowed northward up the lower part of the present valley of Mill Creek. Thence it continued northward, up the Mill Creek Valley, to a point about three miles south of Hamilton, where it was joined by the Miami River. During this period of drainage the valleys were cut to depths of several hundred feet below their present floors and were then refilled and again partly excavated. The principal materials of the fill are sand and gravel, interbedded with more or less continuous lenses of impervious till and clay, which, where present, act as barriers that confine the water in the underlying gravels under artesian pressure and also prevent surface-water from reaching these gravels.

Progressive decline in water-levels in wells in this area has been noted for many years and has been greatest in the industrialized centers around Norwood, St. Bernard, Lockland, Hamilton, and Middletown. The water-level in Norwood, where the glacial fill is about 240 feet thick, is reported to have been about 110 feet below the land-surface in 1907; but the water-level on December 1, 1938, in a well at the waterworks of the City of Norwood, was 191 feet below the land-surface. A well at the plant of the Procter and Gamble Company, near St. Bernard, at the junction of the Norwood Trough and Mill Creek Valley, is reported to have flowed at the surface in 1883; but the water-level in an abandoned well at the same locality on December 1, 1938, was about 89 feet below the land-surface. The water-level in the wells of the Stearns and Foster Company, in Lockland, is reported to have been 27 feet below the land-surface in 1883; but the water-levels in the present wells of the Company are about 119 feet below the land-surface. Smaller declines have been reported in Hamilton and Middletown.

Several causes for this great decline in water-levels have been suggested. The pumpage has been greatly increased with the growing industrial development and also as a result of the draining of the Miami and Erie Canal, which forced industries that formerly used water from the canal to pump their water from wells. On the other hand, changes in drainage now allow the storm-water to run off rapidly and afford less opportunity for it to seep into the ground; and during the past decade the deficient precipitation has doubtless resulted in a definite diminution in the annual rate of recharge.

Estimates of pumpage from wells in the Norwood Trough, Mill Creek Valley, and the valley of the Miami River have been compiled from data obtained from about 69 industrial and municipal well-owners. The average daily pumpage as of December 1, 1939, from each area is shown as follows:

Norwood Trough	3.5 million gallons per day
Mill Creek Valley between St. Bernard and Hartwell	6.0 million gallons per day
Wyoming, Lockland, and Reading	6.5 million gallons per day
Mill Creek Valley, north of Lockland	1.0 million gallons per day
Hamilton, New Miami, Rockdale	23.7 million gallons per day
Middletown, Amanda, and Excello	25.6 million gallons per day

There is thus a total pumpage from the area, exclusive of the downtown-section of Cincinnati, of about 66 million gallons of water per day, or almost 24 billion gallons per year.

Precipitation at Cincinnati during 1930, 1934, and 1936 was well below normal, and the accumulated deficiency in rainfall at Cincinnati from January 1, 1930, to July 1, 1936, was slightly more than 35 inches. During July, 1936, to July, 1939, precipitation has been in general above normal so that, by July 1, 1939, the deficiency accumulated since January, 1930, was only about 13 inches. Precipitation during the last five months of 1939 has been below normal.

Measurements of water-level have been made wherever possible and in certain observation-wells at weekly or monthly intervals. Fifteen automatic water-stage recorders which continuously record the fluctuations of the water-level have been installed on wells, largely in areas of heavy pumping, where the fluctuations of water-level caused by pumping of near-by wells are so great that individual measurements are of little value.

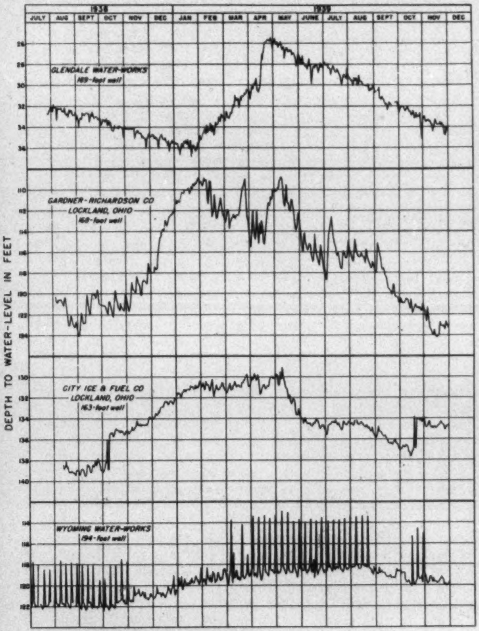

PLATE I – GRAPH SHOWING LOWEST WATER-LEVEL REACHED EACH DAY IN FOUR WELLS IN HAMILTON COUNTY, OHIO, JULY 1, 1938 TO DECEMBER 1, 1939 AS DETERMINED FROM AUTOMATIC WATER-LEVEL RECORDER-CHARTS

The water-level in a well fluctuates continuously, moving up or down in response to changes in precipitation, evaporation, transpiration, atmospheric pressure, and pumping from near-by wells. The magnitude of such fluctuations may range from several hundredths of a foot to many feet. The water-level in an unused deep well at the Glendale Waterworks, about 60 feet from a pumped well, fluctuates as much as seven feet several times a day, as pumping from the other well is successively started and stopped. In this case, a single measurement would be of little value in determining the trend of the water-level.

In order to determine whether the water-level is rising or falling during a period of several months or years, in areas where near-by wells are being pumped, it is necessary to have a continuous record of the water-level. To eliminate daily fluctuations, the lowest water-level reached each day is determined from the recorder-chart and is plotted on a yearly water-level curve (see Plate 1).

The curve of the water-level at the Glendale Waterworks shows that on December 1, 1939, the water-level was about

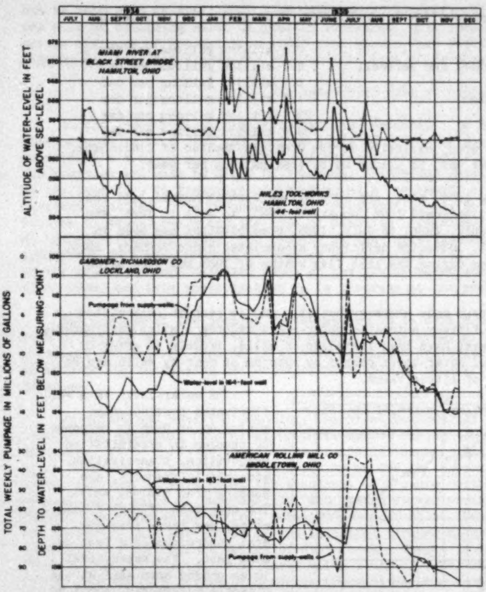

PLATE 2 - GRAPHS SHOWING RELATION BETWEEN FLUCTUATIONS OF WATER-LEVEL IN OBSERVATION-WELLS AND PUMPAGE FROM SUPPLY-WELLS AT GARDNER-RICHARDSON CO., LOCKLAND, AND AMERICAN ROLLING MILL CO., MIDDLETOWN, AND BETWEEN WATER-LEVEL IN WELL AT NILES TOOL-WORKS AND STAGE OF MIAMI RIVER AT BLACK STREET BRIDGE, HAMILTON, OHIO

half a foot higher than on the same date a year ago and was about two feet higher than the low level in January, 1939. This rise in water-level indicates an increase in ground-water storage and is caused in large part by recharge. An interesting feature of the curve is the sharp rise of more than four feet during April 15, 16, and 17, 1939, when the surrounding fields were flooded.

The curve of water-level in an abandoned well at the plant of the Gardner-Richardson Company in Lockland shows that the water-level in this well is greatly influenced by pumping from near-by wells. The rapid rise of water-level in December, 1938, was due in large part to a great reduction in the rate of pumping from the surrounding wells and probably to a lesser degree to recharge from precipitation. The one-year record from September, 1938, to September, 1939, shows a rise in water-level of almost five feet. However, during the last three months, pumping from near-by wells has been increased, precipitation has been below normal, and the water-level has declined. On December 1, 1939, it stood about three feet lower than on December 1, 1938, but about one foot higher than the low level of 1938. The water-level in a well at the Lockland plant of the City Ice and Fuel Company shows a sudden rise of three feet in early October, which was caused by the cessation of pumping from wells at that plant during the winter. Pumping for the summer of 1939 was started about May 8, and was continued until October 15, 1939. The water-level on September 1, 1939, was about three and a half feet higher than on September 1, 1938. Because of the decline in water-levels during the past two months, on December 1, 1939, the water-level was half a foot lower than on the same date in 1938.

The large fluctuation each week shown by the curve of the water-level in an observation-well at the municipal water-plant of the Village of Wyoming is caused by cessation of pumping on Sundays and consequent rise in water-level. During the winter, pumping is continued until early Sunday morning, and as the lowest daily water-level is plotted, the large fluctuations are not shown on a curve of this type during this period. On December 1, 1939, the water-level stood 1.7 feet higher than on December 1, 1938. As pumpage from this plant was greater during the fall of 1939 than during the same period in 1938, the rise in water-level shows an increase in ground-water storage primarily due to recharge.

In many well-fields the principal fluctuations of water-level in an observation-well are closely related to changes in rate of pumping from near-by wells. Such fluctuations obscure rises in water-level caused by recharge and it is often difficult to determine how much of the rise is due to changes in rates of pumping and how much to recharge. The total weekly pumpage and the lowest water-level reached each week have been plotted together in Plate 2. At the East Side works of the American Rolling Mill Company in Middletown, the water-level in an observation-well, as shown by the solid line, declined steadily from August 1, 1938, to April 7, 1939. When the weekly pumpage, as shown by the dashed line, was materially decreased during April and May, the water-level rose about six feet. Increase in pumpage during June was accompanied by a decline of water-level. In July, when the total weekly pumpage was about one-third of normal pumpage the water-level rose continuously a total of about 17 feet. When pumpage was increased the water-level declined rapidly and on December 1, 1939, it reached the lowest stage for the period of record up to that time, that is, 111 feet below the surface.

The same curves have been plotted for the plant of the Gardner-Richardson Company at Lockland, where the fluctuations of water-level bear even closer relation to the weekly amounts of pumpage from wells.

Water-levels in wells may also be influenced by high stages in near-by streams. The solid line of the upper curves in Plate 2 shows the water-level in a well at the Niles Tool works in Hamilton, and the dotted line represents the stages of the Miami River as determined by weekly measurements of water-level from the Black Street Bridge. The two curves show similar fluctuations, although it is interesting to note that the ground-water level remains several feet below river-level, presumably because of pumping from wells in the vicinity. The water-level in this well on December 5, 1939, was about three feet lower than on the same date in 1938.

Any adequate quantitative study of ground-water supply in an area must be based on data accumulated over a considerable period of years, and the longer the record, the more accurate are the resulting conclusions. Any deduction as to the perennial daily water-supply available, or as to the general trend of the water-level made from measurements taken during a brief period when recharge conditions may be far from normal, are likely to be premature and to lead to erroneous conclusions. However, bearing in mind certain qualifying factors, tentative conclusions regarding the ground-water supply in Butler and Hamilton counties may be reached.

During July 1, 1938, to July 1, 1939, business activity in Butler and Hamilton counties was at a relatively low stage and the rate of pumping was less than normal for the full operation of the existing industries. Precipitation for the period was about nine inches above normal and conditions were especially favorable for ground-water recharge in the first part of 1939. Under these conditions, the recharge of the water-bearing beds was roughly equal to the amount of water pumped during this period. Since the recharge was probably greater than normal, an important question arises as to the average amount of recharge during periods of more normal precipitation.

The record shows that in the seventeen-month period of investigation, there has been little, if any, permanent decline in water-level in Butler and Hamilton counties, because of curtailed industrial pumpage and abundant precipitation. However, if industrial pumpage is much increased, or dry years occur, the water-levels may decline rapidly. The investigation has shown the encouraging fact that the pumped water is not merely taken from storage but that there is large recharge in seasons of abundant precipitation. On the other hand, it has shown that the present water-supply is uncomfortably close to the perennial safe yield, with inadequate reserve for future expansion, and that additional sources of supply should be developed.

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U. S. Geological Survey,
Washington, D. C.

THE HYDROLOGIC PROGRAM OF THE WEATHER BUREAU

Thomas S. Southwick

Within recent years the hydrologic cycle has been subjected to intensive investigation. This has been partly the result of the natural evolution of the applied science of hydrology, from its older and empirical basis through the development of more rational and precise methods, to its present quantitative state.

A significant factor has been the intensified public interest in flood-control. Disastrous floods are of recent memory and now, under the impetus of the Flood Control Act, a program of flood-control is being actively engaged in by the Departments of War and Agriculture. Of special significance in the program is the attention being given to runoff-conditions in upstream-areas as well as the control of flood-waters downstream. This phase, which is generally under the flood-control agencies of the Department of Agriculture, aims at measures which will not only make for conservation of the soil and forest-cover but which will act to decrease the peak-flow downstream.

The Weather Bureau is closely concerned with all phases of the hydrologic cycle. Recent contributions to our knowledge of the characteristics of air-masses associated with flood-producing storms and the expansion of observations into the third, or vertical, dimension permit us to forecast the intensity and extent of rainfall to a degree hitherto not possible. The Weather Bureau is making gratifying progress on the problem of quantitative forecasts of precipitation.

One of the basic duties of the Weather Bureau is the forecasting and issuance of warnings of floods. Accordingly, the Bureau has concerned itself with the newer developments in gathering hydrologic data and in perfecting flood-forecasting technique. This is especially true for those upstream-areas where runoff so closely follows rainfall. In these upper reaches the forecaster must base his forecast on rainfall rather than antecedent stream-flow.

In addition to this long-established service, the Weather Bureau is now engaged in two hydrologic programs, namely, (1) the analysis of flood-producing storms, and (2) the acquisition of basic hydrologic data.

The first-named project, under the Hydrometeorological Section of the Bureau is being carried out in cooperation with the Corps of Engineers and the Flood Control Advisory Committee of the Department of Agriculture. Definite areas for study are assigned the Section by the cooperating agencies. Outstanding storms are analyzed; they include not only the storms which have occurred over the area in question or near-by, but also those more distant storms which can, with reason, be transposed over the area. Synoptic weather-maps are constructed, and the storms analyzed in the light of the latest knowledge of air-masses. Transposed storms are modified to conform to the meteorological and orographic characteristics of the area in question.

From a study of the air-masses and fronts associated with these storms and the meteorological conditions which can reasonably be expected to occur over the storm-area, a synthesized maximum storm is developed. This maximum storm is then considered in the design of flood-control works.

The second project, under the Hydroclimatic Section of the Bureau, is concerned with the collection, tabulation, and dissemination of basic rainfall-data, and is being carried out in cooperation with the same Agencies cooperating in the first project. The effectiveness of recent advances in hydrology has been hampered by the lack of adequate precipitation-data. The need for more data is twofold, namely, (1) a greater number of rainfall-stations and (2) more records of intensities of precipitation. Such modern techniques as the unit-hydrograph, the infiltration-theory, and storm-transposition depend for their maximum effectiveness upon an adequate knowledge of rainfall-intensities. Effective stream-flow forecasting in areas of rapid concentration demands a knowledge of the distribution of rainfall throughout short periods. The current program of the Weather Bureau is designed to meet this need. It has as its objective a nationwide hydrologic network of recording gages, augmented by non-recording gages, as well as batteries of shielded storage snow-gage units in appropriate regions. This network makes the fullest use of the facilities of the Weather Bureau, and includes approximately 800 weather reporting stations at Weather Bureau first-order and airways-stations, and 6000 climatological cooperative stations. Precipitation-stations operated by agencies other than the Weather Bureau will be incorporated into the network through cooperation with these agencies.

For the administration of the program the country has been divided into ten regions. The Supervisors of the regions maintain close liaison with the Federal flood-control agencies in order to insure that the organization will meet the demands upon it with maximum effectiveness.

The data are reproduced in a monthly bulletin which gives the hourly amounts of precipitation for recording gages and the daily amounts for non-recording gages.

While the undertaking is primarily for the benefit of the Federal flood-control agencies, the precipitation-data as gathered will be of inestimable benefit to all hydrologists. In this connection, it may be mentioned that this program embodies basically the recommendations of the Water Resources Committee of the National Resources Planning Board. As expansion occurs both in the increased number of stations and the lengthening periods of record, we shall most certainly have to revise our concept of the rainfall-regimen for many areas. With increased knowledge of the rainfall-characteristics of an area we shall effect a correspondingly great refinement in our hydrologic researches and the designs dependent upon them.

U. S. Weather Bureau,
Cincinnati, Ohio

THE COMPREHENSIVE FLOOD-CONTROL PLAN FOR THE OHIO RIVER BASIN

Lewis A. Pick

Introduction--The severe flood-losses experienced in the Ohio River Basin during the great floods of 1936 and 1937 aroused the citizens of the affected localities as never before to the realization that comprehensive measures must be undertaken to avoid in the future similar demoralization of the economic life of the region. This manifested interest led to the enactment of legislation by Congress approving the general comprehensive flood-control plan for the Ohio River Basin as submitted by the Chief of Engineers, United States Army, in Flood Control Committee Document No. 1, 75th Congress, 1st Session.

Problems involved in development of plan--The Ohio River Basin lies directly in the path usually followed by cyclonic disturbances as they move from west to east across the North American Continent in the winter and early spring months. For this reason the watershed is frequently subjected to more than normal rainfall during the months of January to March, when infiltration, transpiration, and evaporation are at a minimum and when rainfall-runoff relationships attain their maximum. This is the major factor accounting for the magnitude of stream-flows during these seasons of the year.

Other contributory factors to Ohio River floods are (1) the precipitous slopes of the mountainous region bordering the Basin to the east and southeast, which occasion excessively high rates of runoff in those regions, and (2) the pear-shape of the Basin, which tends to synchronize flood-flows originating in the upper or narrow portions of the watershed with those of the three large tributaries which drain the wide area toward the mouth--namely, the Wabash, Cumberland, and Tennessee rivers. The flood-problems on the Ohio tributaries have also been aggravated by the encroachments on the floodways of buildings, bridges, railroads, highways, and other structures.

Methods of flood-control applicable to Ohio River Basin--All methods ordinarily employed in securing flood-control, or utilized in the prevention of floods, are applicable in some degree to the solution of the flood-problems in the Ohio River Watershed. At certain localities in the headwaters of the tributaries, such as at Johnstown, Pennsylvania, where the steep slopes of the streams and the highly developed state of their valleys render reservoir-control impracticable from an economic standpoint, either channel-improvements or local protection-measures must be utilized. However, in general, flood-control reservoirs, supplemented in some cases by local works, constitute the most economical method in the solution of the tributary flood-problems, whereas, as will be explained later, the principal reliance for flood-prevention along the main stream must be placed in local protection-works such as levees and walls.

The fact that the rainfall which caused the flood of 1937 fell largely in the main valley of the River, coupled with the great magnitude of the resulting runoff, indicates the difficulty and expense of achieving flood-control along the Ohio River proper by reservoirs alone. For instance, in order to maintain the River below the no-damage stage (52 feet) at Cincinnati during the flood of 1937, it would have been necessary to store a total of over 11,000,000 acre-feet of water with storage facilities of 100 per cent efficiency. From a group of widespread tributary reservoirs, computations have indicated that no more than about 20 per cent efficiency can be expected, considering the non-use of some of the reservoirs in consequence of rainfall-distribution. Therefore, five times 11,000,000 acre-feet would need to be provided in the reservoirs. At $37 per acre-foot, the average cost of reservoir-storage above Cincinnati, such a capacity would cost over two billion dollars. The cost of reservoir-control would be correspondingly increased below Cincinnati, since the effect of reservoir-storage diminishes rapidly as the distance from reservoirs to the points to be protected increases.

Procedure employed in developing the plan--The orderly development of a comprehensive flood-control plan for a great watershed such as the Ohio Basin demands that the flood-problems of the individual tributary basins be first considered. Local flood-control plans having been developed for the tributaries, the next step is to determine the effect of these individual tributary plans on the main stream and to determine what modifications in them are justified from the standpoint of main stream flood-control. This having been done, the third step is to establish the supplementary measures along the main river which are necessary to obtain the desired degree of flood-relief. The fourth step is to determine what other phases of stream-improvement may be combined in the comprehensive flood-control plan thus evolved. In this last step, it is important that the economic value of each prospective project be thoroughly analyzed for all alternative purposes, such as navigation, water-supply, sanitation, and hydroelectric power-development, and for those purposes in combination with flood-control, to assure that the project will be developed in the best interests of the general public.

The careful investigation by the War Department's engineers of the possibility for obtaining reasonably complete control of floods along the Ohio River by means of reservoirs or diversions has definitely indicated that such methods alone will be inadequate below the mouth of the Beaver River. Accordingly, the principal reliance for flood-prevention along the main stream must be placed in levees and walls, except in a limited section of the upper River in the vicinity of Pittsburgh.

The general basis for design of these works below Wellsville, Ohio (just below the mouth of the Beaver River), has been the provision of three feet freeboard above the maximum high-water of record. Above Wellsville, the reservoirs required for local flood-control will materially reduce stages so that in this upper section the basis used has been three feet freeboard above the profile of the reservoir-design flood, as reduced by reservoirs proposed above Pittsburgh. This adopted criterion will provide immediate relief from all floods of record at those communities where flood-walls or levees are built.

These local protection-plans do not provide any protection to outlying communities and farms along the River. In order to reduce both the magnitude and frequency of overflows in these outlying regions, it has been found economical and desirable to supplement the levee and wall-plans described in the previous paragraphs by a reservoir-system. Such a reservoir-system will also assure the adequacy of the local protection-projects against overtopping during a possible greater flood than the maximum of record which has been used as a basis for their design.

This supplementary reservoir-system has been evolved on the following basis:

(1) All reservoirs considered meritorious from a tributary standpoint, which would have a material effect on Ohio River floods, have been included in the system.

(2) Sufficient additional reservoir-projects have been added to assure that the system will reduce a flood of the magnitude of a hypothetical "project-flood" to approximately the stage of maximum recorded high-water at all locations. In the selection of these additional projects, preference has been given to those reservoirs which would result in a maximum of local benefits.

Nature of benefits attributable to the plan--The reservoirs in the plan have been designed primarily for flood-control on the Ohio River and its tributaries, but their operation will incidentally afford a reduction in Ohio River contributions to floods on the Mississippi River--a feature which will further assure the adequacy of the flood-control plan for that stream. In addition, they have been planned to provide permanent pools which will be of great value for recreation or as possible wildlife preserves; and the reservoirs may be operated to afford a valuable increase in low-water flow on the Ohio, providing a benefit to navigation, improved water-supplies, and sanitary conditions without sacrificing flood-control storage. Operating in conjunction with the reservoirs already completed in the Basin, the proposed reservoirs, with the local protection-works that have been planned, will afford essentially complete flood-protection throughout the urban and highly industrialized portions of the Ohio River Basin. Latest estimates indicate that the benefits of the reservoir-system will exceed its cost by a wide margin, and that a large number of local protection-works are economically justified at this time. As the region further develops, it is expected that additional projects of similar nature will be warranted for construction.

Congress has authorized the initiation and partial accomplishment of this comprehensive plan to the extent of expenditures not to exceed $75,000,000 for reservoirs and $50,300,000 for local protection-works, exclusive of previous authorizations contained in flood-control legislation enacted prior to 1938. The total authorizations under law amount to about $115,000,000 for reservoirs and $88,000,000 for local protection-projects. The reservoirs and local protection-projects to be built under these authorities, except for a few projects specifically mentioned in legislation enacted by Congress, are to be selected and approved by the Chief of Engineers.

Procedure in selection of projects--In order that the selection of the projects may proceed on the basis of sound economic and engineering considerations, the War Department has undertaken all types of engineering investigations including the collection of basic meteorological, hydrological, and geological data on a scale never before attempted by any agency, governmental or private, in this region. In the conduct of the investigations it has been found to be in the interest of economy to call upon other agencies of the Federal Government, as well as upon private consulting engineers for assistance. Among the Federal agencies assisting in the studies are the United States Geological Survey on stream-gaging activities, the United States Weather Bureau on meteorological studies, the Department of Agriculture on evaluation of flowage and flood-damages to agricultural lands; and advice has been secured from the Bureau of Mines in regard to purchase of coal-lands and other mineral resources affected by projects.

Often it has also been found desirable to employ a Board of nationally known consulting engineers to help our engineers in the design of major projects, such as, for instance, the recently completed Tygart Dam. This practice not only results in facilitating the design and construction of the projects, but also has enabled the War Department to keep abreast of the most recent developments in the art of engineering design and construction as evolved by private interests.

Present construction-program--While selection of projects for construction under the present limited authorizations of Congress has not been completed, a sufficient number of projects has been tentatively selected to absorb expenditures of from 40 to 50 million dollars annually during the next two fiscal years. It is anticipated that selection of additional projects to use up the entire expenditures on the comprehensive plan now authorized by Congress will be made in ample time to permit the present program to be entirely completed by about 1945, provided the necessary funds are appropriated.

Some of the major works now under construction are worthy of mention. These include: An extensive channel-improvement project at Johnstown, Pennsylvania; five large reservoirs on tributaries of the Allegheny and Monongahela rivers above Pittsburgh; extensive wall- and levee-systems for the Ohio River cities of Huntington, West Virginia, Ironton, Ohio, Tell City and Evansville, Indiana, and Paducah, Kentucky; and a channel-rectification project for Middlesboro, Kentucky. Other jobs are scheduled to start in the near future so that, by the end of this present fiscal year, we hope to have under contract flood-control works in the Ohio River Basin aggregating in cost in excess of $100,000,000.

Corps of Engineers, U. S. Army,
Ohio River Division, Cincinnati, Ohio

HYDROLOGIC ASPECTS OF THE MUSKINGUM WATERSHED CONSERVANCY DISTRICT

C. C. Chambers

The Muskingum Watershed Conservancy District, with headquarters at New Philadelphia, Ohio, was organized under the Conservancy Act of Ohio by a decree filed June 3, 1933, in the Clerk's office of Tuscarawas County, Ohio. It was created for the purposes of preventing floods and of conserving water for beneficial uses, and incident thereto, the development of water and other natural resources, river-regulation, purification of streams, prevention of soil-erosion, and forestation and recreational developments. The territory covered by the District is practically the entire 8038 square miles of drainage-area of the Muskingum River and its tributaries. All or a part of 18 of the 88 Ohio counties are included in the District.

The first step to meet these objectives has been the construction of a series of 14 flood-control and storage reservoirs to hold back and store flood-waters. There are four flood-control or detention reservoirs with a total capacity of 739,600 acre-feet of flood-control storage. The other ten are combined flood-control and storage reservoirs with a total capacity of 799,600 acre-feet. Of the latter, 587,800 acre-feet are reserved in the top for flood-control, with the remaining 211,800 acre-feet in the reservoirs to be used for conservation of water by storage for water-supply, industrial use, the production of steam, maintenance of sanitary conditions of stream-flow, irrigation, recreation, fisheries, and other uses.

In general these combination basins and reservoirs are on tributary streams. Nine of them have drainage-areas of 300 square miles or less at the dams. The other, namely Wills Creek, has a drainage-area of 844 square miles. The total water-area in the ten reservoirs when filled to conservation storage-levels is 16,000 acres, ranging in the individual reservoirs from 420 to 3550 acres.

The outlet works at all of the dams have regulating gates so that flood-waters in excess of the capacities of the stream-channels at critical points below the dams can be temporarily held in the reservoirs and then drawn out as rapidly as the excess water can be safely carried away by the stream-channels, after the natural flood-crest has passed. The amount of gate-opening at any dam can be changed during the period of any flood to meet the changes in head of water in the reservoirs above the dams and other changing conditions. The gate-regulation also makes the operation of each reservoir more flexible as a part of the whole system, and in meeting variations of rainfall and other conditions in different parts of the drainage-area.

Hydrology in its broadest aspects treats of the properties, laws, and phenomena of water, of its distribution and circulation, and of the effect of these on society. As nearly all of the objectives of the Muskingum Conservancy District have to do with the control of water and its adaptation better to meet the needs of man, it follows that hydraulic engineering, of which hydrology is the basic consideration, has played an important part in the development, construction, and operation of the project.

All sciences are so interwoven that no one science can be pursued without trespassing to some extent on others. Therefore the hydraulic engineer must seek information from the geologist, meteorologist, geographer, agriculturist, forester, sociologist, and others. The help of all of these was essential in the study of the hydrologic problems of the Muskingum Area.

The northwest part of the District was visited by glaciers that left their influence on the topography and on the geological formations for some distance below the surface, whereas the southeastern part of the Area is unglaciated. These wide variations in the geological formations introduced different conditions of surface-flow, infiltration, and subsurface movement of water. With the Area reaching from the watershed-line on the north, between the Ohio River drainage and the drainage to the near-by Great Lakes, down to the Ohio River itself on the south, there is sufficient spread in territory for possible variations in atmospheric conditions to influence temperatures and precipitation. These are things in which the meteorologist is interested. The physical and civil geography influenced the study of the problems. The agricultural history of the Area and probable future land-use affected the estimated movement and use of water, and the same was true of the forest-covered areas. Often projects developed by the hydraulic engineer and these other scientists have failed because the sociologist was not consulted. Sometimes it is a greater task to adapt society to the benefits of an undertaking than to build the project itself. The only reason for the construction of any engineering project should be that, as a result of the construction, sufficient benefit will accrue to some individual or group of individuals to warrant the expense of the construction.

There will be opportunity in this paper only to outline briefly some of the hydrologic problems that were presented in the development, construction, and operation of the project, and a general idea of their solution.

Water first influences man when it is precipitated through the atmosphere on the surface of the Earth. The first hydrologic aspect of the Muskingum Conservancy District was the attempt to find a rational answer to the question of how much precipitation will there be over this Area, and what will be its distribution. Our predictions for the future must be guided largely by our knowledge of the past. The longer the history of rainfall-records and the more thorough their recording and interpretation, the more accurate will be the predictions. Reliable records for the particular area under study have great weight, but this Area has records for only a comparatively short time. General rumors and reports on extreme high and low precipitations extend back little beyond a century and a half, while official measurements only began with a station established in 1818 at Marietta at the confluence of the Muskingum with the Ohio River. The next station was at Granville in 1854. Any continuous record over the Area has been for only half a century, and for that period the drainage-basin has been covered by a scant dozen scattered stations. In 1933 there were only 20 precipitation-stations in the Muskingum Area, and there were seven others just outside the limits of the watershed.

For the past half-century, the average of the annual mean precipitation was a little over 39 inches for the Muskingum Area, with a maximum in excess of 56 inches for the year 1890 and a minimum of 25.3 inches for the year 1930.

One of the first problems confronting the planners of the Muskingum Project was to determine for what kind of storms, and for what conditions the flood-control part of the reservoirs should be planned. The greatest general flood for the Muskingum Area of which we have detailed knowledge was that of March, 1913. It is important to know, however, that there are records on some tributary streams of floods in excess of the flood of 1913.

Intelligent use of every available record here and elsewhere, while not giving all that could be desired, gave sufficient information so that fairly accurate predictions could be made as to the probable amount of precipitation to be expected, its distribution both as to seasons and over the Area, and the probable maximum and minimum stream-flows. It was determined that the reservoirs should be designed with capacities and with a plan of operation so that they would be filled to spillway-level--but not beyond--in case of a storm totaling ten inches of rainfall over the entire Area and distributed over five days in the same proportion as the rainfall that actually occurred during the storm of March, 1913. This was given the name "Official-Plan Storm". It was recognized that storms with less total rainfall but concentrated in a shorter period of time will cause just as large floods as the Official-Plan Storm, and that the reservoirs need to be designed for them also.

The next problem was to determine a proper relationship between the amount and distribution of the rainfall and the amount and distribution of the flood-runoff resulting from that rainfall. As near as can be determined from the available records, 86 per cent of the rainfall that fell on the Muskingum Drainage-Area above Zanesville was transferred to runoff to be carried away by the stream-channels during the period of the flood of 1913. It is believed that the ground can be so saturated with water at the beginning of a storm that, even though there be no snow or ice on the ground, and regardless of the vegetation-cover and the use of the lands, this relationship can be as high as 90 per cent. Therefore the estimated flood that would result from the Official-Plan Storm was computed on the basis that 90 per cent of the ten inches of rainfall or nine inches of runoff from the drainage-area would be the volume of the flood-water to be taken care of. It then became necessary to distribute this volume of water and develop discharge-hydrographs at many points along the streams. Distribution-curves were derived for some of the individual drainage-areas by computations based on the records of storms for which both rainfall and runoff are accurately known. This method of determining the distribution of runoff for a large storm from the records of a smaller storm is not applicable for large drainage-areas in excess of 1000 square miles, and the distribution-graph method was supplemented, therefore, by other recognized methods of developing flood-hydrographs for the maximum expected storms.

The Official-Plan Storm of ten inches over the entire drainage-area is 36 per cent greater than the average rainfall for the flood of March, 1913, above Zanesville, and the estimated peak-flood discharge at Zanesville for that storm is 22 per cent greater than the peak-discharge of 1913. Although storms in the Muskingum Area equivalent to the Official-Plan Storm and floods equivalent to its resulting flood may on the average occur only once in a century or more, it is quite probable that such a storm might happen at any time.

The selection of a suitable site for a flood-control reservoir must have careful study by the hydraulic engineer. Some sites considered for the Muskingum Project were abandoned when studied from the hydrologic aspect, even though storage could be provided at less cost per acre-foot than at some of the sites finally adopted. In certain cases it was found that the natural valley-storage at flood-times in the sections of valley to be occupied by the reservoirs was 80 per cent or more of the gross storage in the reservoir so that the net increase in the storage was less than 25 per cent more than what would have been stored in the limits of the proposed reservoir under natural conditions. In other cases the time of crest of flood-waters from the tributary ordinarily preceded the crest in the main stream so that its control would have little effect on reducing maximum flood-stages in the larger streams. At other sites the crests of the flood-waters from the drainage-areas above them ordinarily arrived after the main flood in the larger streams. In the system finally adopted, some of the reservoirs are on tributary streams where flood-crests travel slowly, and, to be of any help even to local communities, the flow at the dams must be entirely shut off during the first part of the flood-period. Such reservoirs will give little, if any, benefit toward flood-reduction on the Ohio River for floods resulting from a single storm, but the Muskingum Project was originally planned to provide benefits to the local communities in the Muskingum Area, and benefits to the Ohio River were entirely incident thereto. The proposed transfer of the Project to the United States may reverse this original idea.

Proposed sites for the Muskingum plan had to be considered both from individual standpoint and with respect to their place and value in the system as a whole. The proposal that some of the reservoirs should provide water-conservation storage in addition to flood-control storage also influenced the selection of sites. Many different combinations of reservoir-sites were studied, and various types of floods were routed through these different arrangements before the system which seems best designed to meet the objectives of the District was finally adopted.

The next important hydraulic problem involved the design of the structures, especially with respect to spillway-capacities for the earth-dams. The approach to this problem must turn from the greatest probable condition to the greatest possible condition and under the most adverse circumstances imaginable. Hence the spillway-capacities were required to meet the severest kinds of criteria, including assumptions that man would entirely fail in any orderly operation of the regulating gates.

Many other hydrologic aspects entered into the detailed design of the structures at the dams. Nearly all of the dams are earth-fill with concrete outlets. Hydraulic features of the outlet structures were approach-channel, intake, main conduits, stilling basin, and outlet channel. The hydraulic designs followed commonly accepted methods supplemented by model-tests. Some of the earth-dams consisted of an impervious embankment resting on a continuous pervious horizontal stratum. Hydraulic studies, by which the amount of seepage that would occur for such construction was estimated, dictated the requirements of design necessary to increase the lines of travel for the seepage water so that the seepage could be brought below harmful amounts. These were also supplemented by model-studies. For the concrete dam and other concrete structures the customary hydraulic studies for uplift and similar design problems were required. In a few cases on the Muskingum construction, attempts were made to allow the laboratory to specify the amount of water-content for the earth-materials employed in the embankments without using the benefit of engineering experience in the application. In some of these instances the results were unsatisfactory.

Although the hydrologic aspects of the planning, designing, and the construction of the Project were of primary importance and of great interest, the hydrologic aspects in connection with the plans for operating the Project and the actual operation have introduced unique and interesting problems of no little significance.

The flood-control part of the Project was put into operation July 1, 1938. Although there have not been any very large flood-flows since that time, there have been sufficient to show that the hydrologic planning for the operation will prove just as interesting as anticipated. It is believed that the Muskingum Project is not equaled in size and complexity by any other operating flood-control project of its type. For successful operation it must be guided by a central control to which point all information pertinent to the planning is dispatched, and from which the operating plans must be issued. It may be likened to a dispatching system where all influencing conditions may be changing constantly. It is necessary that complete information be available at all times on ground-conditions, rainfall, and stream-stages for the entire drainage-area; and means must be provided for collecting all such information quickly when needed, and there must be a system set up to interpret the information promptly in order that predictions and forecasts can be made in time to develop an operating plan from the beginning of a storm.

Here are some of the questions that must be answered: What is the condition of the ground at the beginning of a storm as to water-content, snow or ice on the surface, and frost? How much rain has fallen, how is it distributed over the drainage-area, how is it distributed as to time, and what are the predictions for rain in the immediate future? What are the stream-stages in the main river and its principal tributaries, and what are the water-levels in the various reservoirs? What is the expected relationship between the amount of rainfall and the amount of its resultant flood-runoff? What is the distribution of that runoff over the area, and what is the distribution of the volume of runoff as to intervals of time? What is the rate of inflow into each reservoir and what volume of water must be passed through each? What is the volume and distribution of the runoff entering the streams from the uncontrolled areas below the dams? The collection and use of all these data cannot wait until a storm is over, but must begin just as soon as sufficient rain has fallen to approach flood-conditions. In summer this may be when an inch or more has descended; in winter and spring seasons it may be less. At intervals of time, probably not more than 12 hours, during the progress of a storm, and after the storm is over and until the reservoirs are emptied of the flood-waters temporarily stored in them, all these data should be collected and interpreted so that new forecasts can be made for stream-flows, and operating plans can be revised, if necessary, to meet the changes in conditions during the last time-interval.

Time will permit only a brief outline of the plans developed for meeting some of these problems. It is recognized that the technique should improve with additional studies and with actual operation experience.

The need for additional rainfall-recording stations has been apparent from the beginning of the Project, but just how close the stations should be in order to give the desired results was not known. Fortunately for the District, the Soil Conservation Service of the Department of Agriculture established the Muskingum Climatic Research Project early in 1937, covering the entire Muskingum Drainage-Area and with headquarters at New Philadelphia, Ohio. They established about 500 observation-stations at four- to five-mile intervals. These are first-order stations equipped with recording rain-gages, accurate instruments for determining velocity and direction of wind, temperatures and humidity, and other climatic observation-equipment including that necessary for evaporation-records at some of the stations. This project has been of great help in planning for the operation of the Muskingum Project. With the area covered so thoroughly with rainfall stations for a period of say five years, an answer should be quite definitely given as to how many such stations are needed permanently to give the desired information on rainfall.

Arrangements now have been made for 18 cooperative Weather Bureau Observers and 17 other observers, including the 14 caretakers at the dams, making a total of 35, to report rainfall promptly to the central control under standing instructions. As reports from the stations indicate that the rainfall is approaching amounts that might cause floods, information is secured from additional stations as may be necessary. The next step is to reduce the known volume of rainfall over the various tributaries and parts of the Area to volume of flood-runoff, for these tributaries and parts. With complete and accurate knowledge of ground-conditions and cover at the time the storm starts, experienced judgment should provide a factor for reducing rainfall to runoff that for most winter and spring storms (which produce the greatest general floods) should be correct within ten per cent. It is less easy to estimate a coefficient for summer storms.

Ways and means are being studied to make accurate estimates quickly. The possibilities of the "Index-Area" method are being examined carefully. This plan tries to relate the stream-flow from a small index-area to the stream-flow from a much larger areal unit by determining from the former a coefficient of runoff which could be applied to a coefficient-relationship curve to get the total volume of flow for the latter. Runoff-results from an index-area can be known or predicted a few hours after the rain has fallen, whereas the runoff-results for the larger area for the same rain will ensue some time later. In this way, working values may in some cases be known from one to four days in advance, but this plan cannot be used safely until accurate and complete comparative records over a period of years are available. A few years ago the Conservancy District in cooperation with the United States Geological Survey established an index-area on a 1.6 square-mile drainage-area near New Philadelphia. It is being used to determine the value of the index-area idea for the Project. Its relationship to larger units can be worked out now more thoroughly than heretofore because of the large number of rainfall-stations of the Soil Conservation Service Climatic Project, and because, through the help of the United States Geological Survey, the system of stream-gaging in the Area has been broadened and improved. In addition, results for index-area studies are being made available from careful observations on small drainage-units in the Watershed and in connection with a Hydrologic Studies Project of the Soil Conservation Service in Coshocton County. The District has been very fortunate to have this project also set up in the Muskingum Area and to have its full cooperation.

Attempts are also being made to estimate runoff-coefficients by infiltration-rates, but one handicap of this idea is that soil- and subsurface-conditions are quite variable over the whole Area.

After the volume of runoff is known, the next step is to distribute this volume as to the time it will pass different points on its travel down the tributary streams and the Muskingum River to the Ohio. This is complex because of the wide variance over the Area in the collection-time and rate of travel in the stream-channels. In the northwest part of the District which has been covered by glaciers, the topography is less rugged than in the southeast part, but the younger stream-channels have steeper slopes so that flood-crests travel more rapidly than in the older channels in the southeast where the slopes are flat. For instance, the time of travel for flood-waters from the Senecaville Dam to Marietta is from five to seven days depending on the type of storm.

Attempts are being made to eliminete some of the limitations of the distribution-curve for use in predicting stream-hydrographs that existed at the time the Project was designed. At the present time there are 32 stream-gaging stations at critical points on the streams in the Area; all are being carefully maintained and operated, so that now accurate records of the volumes of the stream-flows in various streams are available. Added to this is the detail and accurate information on rainfall supplied by the Soil Conservation Service Project. With precise records of these two important factors, distribution-coefficients are being developed for the tributary streams, and the data thus assembled will in time permit forecasting for points on the main streams, from one to four days in advance, the stages that may be expected from the rainfall up to the time of the forecast. Studies were first concentrated on the uncontrolled areas below the dams, and now fairly dependable distribution-coefficients for 12-hour periods are available for these areas.

With the volume and distribution of runoff from the uncontrolled areas known, and the volume of runoff passing through the reservoirs known, it can be quite definitely established how much water needs to be temporarily held, and how much can be permitted to pass, so that the water passing the dams, when added to the runoff from the uncontrolled areas, will not introduce flood-conditions. Then there is the problem of distributing the total allowable discharge at the dams between the 14 reservoirs in the system.

It may be necessary to vary the operating plans for different seasons of the year. During the growing-crop seasons there may be valuable crops in the valleys below the dams that need to be protected by one operating plan, while during other months, when the larger floods occur, a different plan may be desirable to protect improved properties in the flood-plains.

There have been other hydrologic aspects in connection with the design and operating plans for the water-conservation parts of the storage in some of the reservoirs. What use can be made of this water for water-supply, industry, stream-regulation, and other uses, without too great a conflict with recreational and similar uses? It has been found that benefits for these other uses can be provided from conservation-storage, except during extremely dry years, without reservoir draw-downs of more than three to five feet. During extreme years, which occur very rarely, the recreational and related uses may have to make some sacrifices for the other uses if the latter are more important. At some of the reservoir-sites, by careful planning, a total storage almost double the flood-control requirements at the site could be provided, so as to include the conservation-feature, at a very small increase in cost.

Hydraulic studies have also been made as to the probable rate of silting of the reservoirs. It is believed that silting of the detention-reservoirs that are used only for flood-control will not be serious, for all the silt-load carried during normal flows will pass on downstream as heretofore, and during flood-times the excess flood-water is only temporarily detained and will not often be held long enough to deposit the full silt-load. At some of the combination-reservoirs the silting problem may be greater because all the inflow, both during normal and flood-times, must pass through a body of stored water, and a large part of the silt-load will be deposited in the reservoir.

The Soil Conservation Service has further cooperated with the District in establishing controls for making accurate observations on the rate of silting from the very beginning of the operation. Extensive studies are being made to determine what effects will be made on the rate of silting by different land-uses on the drainage-areas above the reservoirs. It is believed that ideas on the silting problem, as well as on many other similar problems, may become badly distorted or exaggerated if results observed on small experimental areas are applied indiscriminately to areas which are much larger in size, or in which basic physical features differ.

For instance, the rate of silting in the O'Shaughnessy Reservoir on the Scioto River above Columbus has been cited as a measure of expected siltation in the Muskingum reservoirs. And it has been stated that, as the former has been silting at the rate of 0.7 per cent of the original storage per year, it follows that the conservation-storage in Muskingum reservoirs will be over one-third filled with silt in 50 years. Although there are many variables to be taken into consideration, one important factor in the rate of reservoir-silting is the ratio between the storage in the reservoir and the area draining into it. The O'Shaughnessy Reservoir had an original storage-capacity of about 17 acre-feet per square mile of drainage-area above it, in comparison with several of the Muskingum reservoirs that have 400 acre-feet or more of conservation-storage per square mile of drainage-area, or a ratio of storage- to drainage-area about 23 times greater than the O'Shaughnessy. Of course, there may be other minor factors that tend to accelerate silting in the Muskingum Area if all other conditions were the same.

One thing that has been impressed upon the minds of those closely associated with the hydrologic problems of the Muskingum Watershed Conservancy District Project is that there are plenty of opportunities to go wrong in the hydraulic design of flood-control reservoir-systems on comparatively large drainage-areas, and it is not a job that can be planned piecemeal. The best, or even a satisfactory, plan cannot be found for flood-control of our larger rivers until thorough study has exhausted all the possibilities of different types of work and different combinations of the various types. And one wonders if some projects that are now started, or about to proceed, have had that thorough study. Those who first saw the possibilities of reservoir-control as worthy of consideration in any comprehensive flood-control system for our rivers are convinced from experience that it still has a place. However, they probably are among the first to recognize its limitations, for reservoirs cannot be the entire cure for all our flood-problems. They fear that improper use will discredit the whole idea. During recent years construction of reservoirs and other ideas for control of floods that have a popular appeal have been greatly dramatized in the public mind. Has the hydraulic engineer allowed this to influence his thinking? If so, it is high time that he put his thinking in order. It is not always easy to sort out the dependable facts from statements influenced by unscientific thinking. Nevertheless the engineer and other scientists must protect their sciences.

Muskingum Watershed Conservancy District,
New Philadelphia, Ohio

LAND-USE AND FLOOD-CONTROL

Wm. F. Simpson and J. S. Cutler

Flood-runoff originates on the land, hence land-use has far-reaching influence. Misuse or abuse of land results in increased storm-runoff and accelerated soil-erosion. Erosion not only destroys the soil itself but the eroded materials contribute to many other damages.

Soil washed from uplands chokes stream-channels and increases flooding. Flood-damages are increased by flood-borne sediment deposited on rich valley-lands and urban properties. In addition, the silting of reservoirs by the products of soil-erosion reduces the capacities for storage-purposes. Ground-water supplies are affected because eroded soil has a lower capacity to absorb rainfall.

The control of soil-erosion is a major factor in soil- and water-conservation, the basis of which is proper land-use. In other words, the first requirement is a plan of sound land-use. In preparing such a plan we must consider the capabilities of the land. Because of soil-types, steepness of slope, degree of erosion, drainage, or other factors, some lands are suited only to the production of woods. Other lands are capable of being used for permanent pastures or meadows. Only the lands best suited to tillage are considered as cultivable crop-lands, and even these are further divided into classes according to the type of management and treatments necessary to control erosion and maintain soil-productivity.

Land-use planning in connection with flood-control has not been carried out to any extent. The Omnibus Flood Control Bill passed by Congress in 1936 authorized the United States Department of Agriculture to participate in flood-control through the application of measures for runoff-control, water-retardation, and soil-erosion prevention on the watersheds of flood-source areas, in conjunction with the well-known work of the War Department for flood-control consisting of reservoirs and levee-systems for downstream protection. Accordingly the United States Department of Agriculture is conducting surveys to develop plans for land-use and related measures for flood-control in several selected watersheds.

While soil- and water-conservation and flood-control are definitely related, the determination of the extent of these relations actually constitutes a new field of work. We touch here, only briefly, the major objectives of flood-control surveys as conducted by the Department of Agriculture and discuss general methods and procedures. The survey on the Muskingum River Watershed in eastern Ohio, now nearly completed, serves as an example. Its general objectives are as follows: (1) Determination of extent of flood-problems; (2) analysis of factors contributing to these problems; (3) preparation of plans for remedial measures; and (4) calculation of costs and values of benefits.

(1) Determination of extent of flood-damages--Detailed economic surveys of flood-damages were made and the damage-areas were outlined on maps. Types of damages, such as losses of crops and livestock, repairs and replacements of highways, railroads, and bridges, and damage to other rural and urban properties were obtained and appraised, using local sources of information. Appraisals were for specific floods in order to determine the flood-stage-damage relationships. With flood-stage frequency-curves prepared from stream-flow data, these damages were calculated on an annual basis. In the Muskingum River Watershed annual flood-damages occurring in isolated areas not affected by the reservoir-system have been found to exceed $200,000--largely agricultural and subject to material reduction by watershed-control measures.

Measurements of recent deposits of silt, sand, and gravel in valley-lands and stream-channels were taken from sample borings to study sedimentation. Stream-bank cutting and scouring of valley-lands by floods were measured using aerial photographs. The actual width and length of stream-banks cut away by flood-flows since 1935 was measured by using these photographs. Depths of these areas were measured in the field and total volume of soil removed was computed. In one secondary tributary area of about 100 square miles, the area of land adjacent to streams removed by stream-bank erosion since 1935 was estimated at more than 26 acres or a total volume of 53 acre-feet. On an annual basis this is 5.3 acres, or ten acre-feet of soil per year, deposited in stream-channels or bottom-lands.

Sedimentation-studies include measurements of deposits in reservoirs, through the use of a spud-bar. After systematic spudding over an area the volume of sediment can be computed and the percentage of loss in capacity obtained. Measurements in the 14 new reservoirs of the Muskingum Valley at intervals of three to five years are planned by the Soil Conservation Service. Sedimentation-damages to highways, railroads, drainage-channels, and other works were usually obtained from maintenance-cost records. Sedimentation-damages in the Muskingum Watershed to date amount to approximately $100,000 annually.

The flood- and sedimentation-damages in terms of money provide the direct economic justification for flood-protection and later serve as a basis for considering the extent of control which may be feasible. Location of the damage-areas indicated the flood-source areas, which then were studied in detail.

(2) Analysis of factors in flood-source areas contributing to flood-problems--In specific flood-source areas detailed studies were made of the hydrology, including stream-channel and flow-characteristics, rainfall- and runoff-relations, and ground-water supplies. Also, closely related studies were made to show the possible effect of existing conditions such as physiographic features, the soils, extent of erosion, and cover-conditions, including woodlands, pastures, meadows, and crop-lands.

In the study of stream-channel and flow-characteristics, profiles of the streams were made and cross-sections of the channels measured at control-points. Considering variations of gradients and channel-dimensions, safe bank-full carrying capacities were computed from slope-area formulas. From stream-flow records found in the United States Geological Survey water-supply papers, hydrographs were prepared for selected floods. Local information on high-water stages and times of occurrence at points upstream from the gages helped to determine the stream-flow characteristics in connection with flood-damages.

Following the analysis of stream-flow, precipitation-records were studied and rainfall-distribution was charted. In the Muskingum Watershed excellent rainfall-records were available from a system of 500 automatic recording rain-gages operated by the Climatic and Physiographic Research Division of the Soil Conservation Service, thus permitting detailed study of distribution, intensities, and duration of rainfalls. By the Horton method of hydrograph-analysis, ground-water flows and channel-storage can be deleted from the total stream-flows, and in this way the watershed-runoff relation to stream-flows for the selected storms is determined.

Under similar intensities of precipitation, runoff-rates are higher during winter when vegetation is dormant and lower during summer when vegetation is active. There is available a great amount of experimental data relative to the effect of vegetation on runoff. These data are now being carefully analyzed, and methods of applying them to large watersheds are in the process of development.

Soils are interrelated with hydrology, having a marked effect on infiltration and water-holding capacity. Previous soil-surveys of Ohio counties and the detailed reconnaissance-survey of the Muskingum Watershed provided information for the preparation of maps for specific areas showing the extent and location of the various soil-types, and slope-, erosion-, and cover-conditions.

Economic studies in specific flood-source areas begin with a review of the history and economic development. Information from the census is studied relative to population, type of agriculture, size of farms, percentages of land in various uses, crop-yields, and production. The object of these studies is to determine the present economic conditions, need for improvement, and extent of change feasible.

(3) Preparation of plans for remedial measures--The next step is to determine the extent to which remedial measures may be applied and to develop plans for operation.

The preceding analysis of factors contributing to flood-problems in specific source-areas began with a study of stream-flow characteristics and proceeded by stages to the actual conditions of the watersheds including economic phases. Thus, the process of analysis of a watershed traces the stream-flow to its source, the land. Planning of remedial measures begins on the land in the watershed and proceeds toward the stream. The objective in flood-control planning is to reduce runoff and erosion on the land by measures which will be beneficial also to the land.

In the Muskingum Survey, planning for remedial measures began with selection of sample farms. Management of these farms was discussed with the owners or operators. Farms were mapped, using aerial photographs. The soils were classified, noting present land-use, slopes, degrees of erosion, and drainage. From this information, plans for improved land-use were prepared for these farms based on capability of the land. Applicable erosion-control measures were included in the plans. The changes these plans would make in the operation of the farms and the effect of the farmer's income were considered.

Data from these sample farms were summarized and expanded to cover the flood-source area under consideration. In this way a general plan of land-use was formulated. Such plans were reviewed by county agents and local land-use planning committees for soundness and acceptability by local people. In addition to plans for the land, sites for possible dams on minor tributaries in flood-source areas were surveyed.

(4) Calculation of costs and values of benefits--Based upon such remedial measures that appear sound, costs were calculated. Costs for such measures include fencing required for land-use changes, tree-planting, gully-control, stream-bank protection, terracing, soil-saving and impounding reservoirs, lime and fertilizers required for pasture, and meadow-improvement and crop-production.

Since most of the items of cost enumerated above represent measures beneficial to the land, as well as contributing to flood-control, it was necessary to evaluate the measures in terms of reduction in flood-damages and benefits to the land through reduction in erosion-damage and maintenance of production. In this way the extent to which Federal Flood Control funds may be used toward putting into operation such measures as were included in the plans was determined. Also, the extent of participation by land-owners or other agencies was determined for benefits other than flood-control.

Flood-control values of remedial measures were obtained by reconstructing selected flood-conditions and assuming proposed measures to be in effect. Beginning with the rainfall-records, runoff was calculated with allowable reductions for increases in woodlands and grasslands, contour-cultivation, and contour-furrows. The resulting reductions in flood-flows were interpreted in terms of reduced flood-damages by correlation with previously determined flood-stage-damage relations. Likewise the effect of small reservoir-storage on stream-flows was evaluated.

As sedimentation-damages are considered together with flood-damages, it was necessary to evaluate reduction in sediment-production through erosion-control and stream-bank protection-measures. Inasmuch as the letter contributes directly to stream-channel filling, the value of reduction in stream-bank cutting through protection may be computed directly and expressed in terms of reductions in damages found in the sedimentation-surveys.

The value of erosion-control on the watershed in reducing sedimentation-damage was determined through comparison of rates of erosion occurring under present and improved land-use and management, as reflected by data from soil-conservation experiment stations. For instance, in one experiment, under cultivation with the corn-rows running up and down the hill, erosion occurs at rates of 60 tons of soil per acre per year, while on the same type of land the soil-loss under contour-strip cropping is only three tons per acre. The difference between the excessive and the controlled erosion indicates the value of strip-cropping toward reducing sediment-production.

Erosion-control benefits to the land were expressed in terms of crop-production. Crop-yields decline steadily as erosion increases. By correlation of yield-data with rates of erosion, with and without control-measures, it is planned to determine the value of erosion-control to the farmer.

Comparison of the total costs with the total benefits determines the economic soundness of plans for improved land-use and related agricultural measures. The values of the benefits to be expected from the remedial measures must exceed the costs if the plans are to be considered economically justifiable.

U. S. Soil Conservation Service,
Region 3, Dayton, Ohio

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